

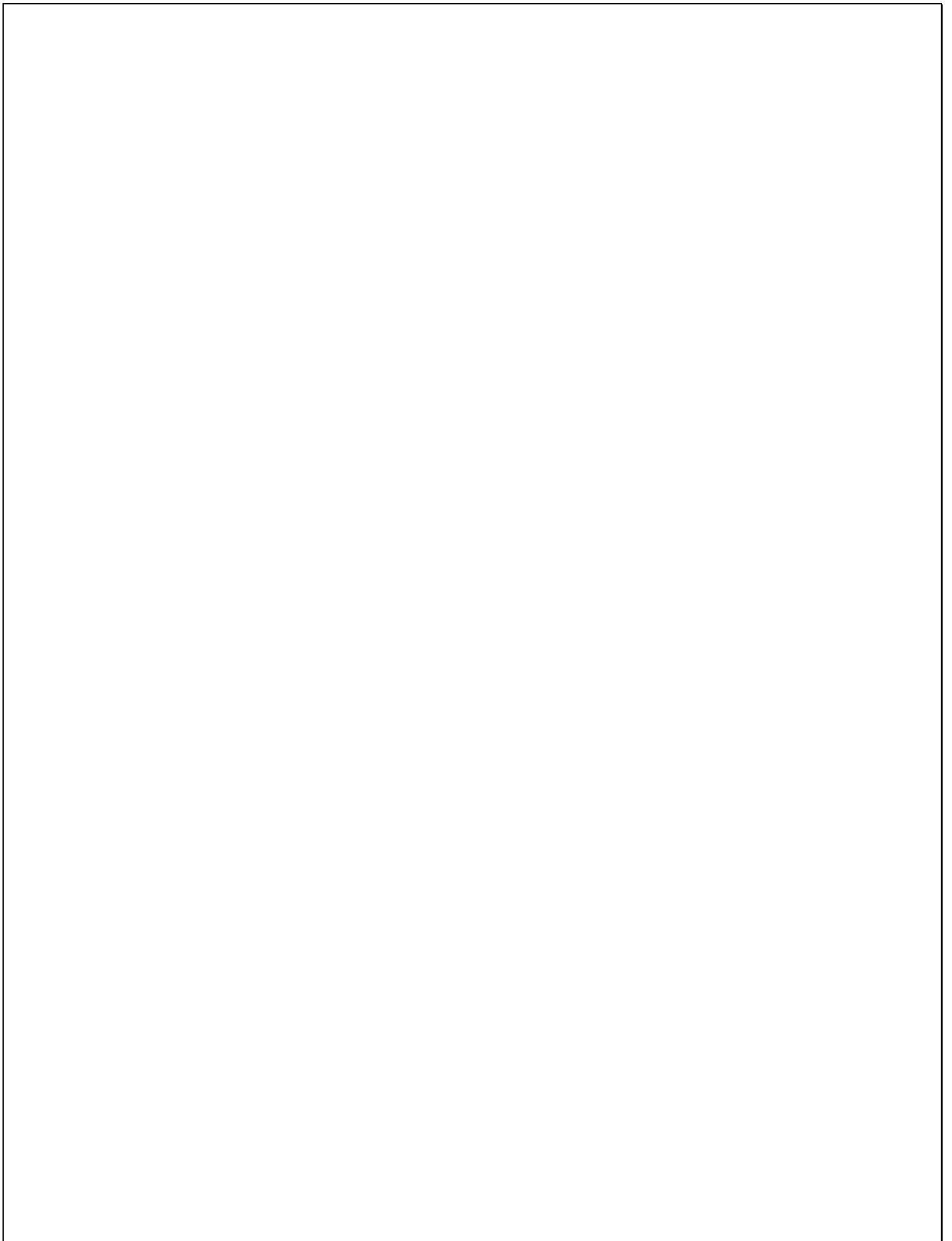


DREYFOUS & ASSOCIATES

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Course Description

Geometry D&A



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## Course Description

The main objectives of the Geometry course are to help students develop high level mathematical skills and to create awareness about the importance of studying mathematics in order to respond to problems and situations that emerge in day to day life. Through the developed content and the strategies and techniques used we intend to bring about a deep understanding of the concepts, as well as the technical skills necessary for subsequent courses and their applications. The way the topics are introduced and presented, as well as the way the mathematical skills are developed through examples and applications, allow students to visualize, understand, and value their usefulness in daily life. The areas and topics discussed in the course include: historical elements of geometry, point, lines, planes, angles and their relationships, triangles and quadrilaterals, polygons, circumference and circles, three-dimensional figures, elements of trigonometry and constructs.

The content takes into account the *Estándares de Contenido y Expectativas por Grado* (Content Standards and Grade-Level Expectations, Puerto Rico Core Standards) of the Department of Education of Puerto Rico (2014) and the United States *Common Core State Standards*. The outline of objectives per lesson takes into account all of the necessary skills and concepts for students to establish connections between the different standards (numbering and operations, algebra, functions, geometry, measurement, and data analysis), into which mathematics are currently categorized. The learning focus is based on conceptual understanding, skill development, and solving mathematical problems, along with the development of critical thinking as a medium for integral development in students.

The course deliberately includes content related to science, technology, and engineering, among other disciplines, with a dual purpose: to help the student see the direct application of what they learn, and visualize the importance of mathematics as a universal discipline that serves society and its institutions. On the other hand, the inclusion of real life problems and situations in each of the topics discussed is intended to awaken an interest toward the study of the discipline.

## Course Structure

The Geometry course consists of thirteen units that are carefully subdivided into several lessons. The amount of lessons per unit depends on the reach and depth with which we discuss and develop different topics. Each unit begins with a series of essential questions to explore and expose students to the situations they will encounter throughout the lesson. They will also find a short video that concretely exposes them to the importance in everyday life of the topics to be discussed. Each lesson has an interactive presentation which is divided into sections in which we expose and explain the topic under study. In each presentation, there are definitions, concrete examples, explanations, simulations, representations with manipulatives, practice exercises, and application of concepts and skills used in daily life.





On the other hand, lessons also include practice exercises, quizzes, extra practice laboratory, homework, self-evaluation and a descriptive log with detailed information for the teacher, as well as a variety of Internet links and other resources.









Some lessons include laboratories that present and reinforce algebraic and geometric concepts, through the use of manipulatives and technological tools like the graphing calculator and interactive activities. The activities are varied and flexible, with the purpose of satisfying the particular needs and interests of each student. The practice and self-evaluation activities aim to make students aware of their strengths and weaknesses in gaining command of the content, with the purpose of having the students gradually take control of their learning process. The teacher, as an essential part of the process, will have the responsibility of stimulating, counseling, guiding, and periodically evaluating the level of learning each student achieves.






The units are made up of the following parts:

## Lessons

Each unit is made up of different lessons, divided by topics, macroconcepts, and skills. Simultaneously, each lesson is made up of five basic elements: presentation or lesson content, documents in digital format (PDF), Internet links, self-evaluation and descriptive log.

- **Descriptive Log.** It's a detailed plan of the lesson. It includes the specific objectives of the lesson, the content standards and grade-level expectations, the teaching strategies and resources, keywords, Internet links, and references. Only the teacher will have access to the descriptive log for each lesson.
- **Presentation (*Lesson Content*).** Each presentation contains a detailed explanation of the lesson's concepts and skills, as per the established objectives. Additionally, they are made up of the following elements that systematically contribute to the development of learning in students:
  - **Examples.** In each section, as skills are developed, we include examples that  explain, step by step, the solution of an exercise or problem, so that students can review concepts and skills.
  - **Practice.** It includes a series of exercises that have been carefully chosen to have students practice the skills and concepts under discussion. Its purpose is to verify the level of learning students have reached before carrying on with other topics and skills. It does not include processes or explanations, only solutions. 
  - **Solution.** It's used to keep the solution to an exercise or problem hidden while students try to answer it. When you touch the button, the solution is displayed. 
  - **Process.** It's a label under which the steps or algorithm for solving an exercise or  problem appear.

- **Proof.** It's a label under which the steps or algorithm for solving an exercise or  problem appear.
- **Calculator.** This includes an explanation of the processes in the use and  management of a calculator in order to solve the exercises in a section. It also connects the students to a virtual graphing calculator.
- **Animation.** It provides access to explanations, processes, or graphics that visually  demonstrate the concepts and skills discussed in the section. They allow us to tend to students' development and conceptual understanding.
- **Definition.** It includes formal definitions of mathematical concepts and processes  that have been mentioned or discussed..
- **Biography.** It includes a short biography of the mathematician or scientist we  credit for the development of a definition, formula, process, or demonstration used in the lesson.
- **Nota.** Under this icon, we discuss common mistakes or reinforce details that must  not be forgotten.
- **Did You know...** It's a section that presents an explanation or situation that  connects aspects of daily life with the skills and concepts discussed. In some cases, this section shows the link between the development of logical thought in human beings with certain skills and mathematical processes.
- **Tabs.** They are on the right side of the presentation and can be red or blue. They  unfold toward the left and provide flow charts, biographies, notes, pictures, explanations, suggestions, reminders, "Did You Know...," or necessary previous knowledge.

- **Incorrect.** It indicates when the student has chosen an incorrect answer in the  practice exercises.
- **Correct.** It indicates that the correct answer has been chosen in an exercise or  practice problem.
- **Picture.** It connects to a particular explanation to a picture that is probably  accessed via Internet.
- **Video.** It helps you access a short video that links the mathematical concept to  daily life.
- **Internet.** It's a direct link to a page that is closely related to the topic. 

Each of the sections included in the presentation are linked to a particular icon that identifies it with its explanation. In the initial presentations of the course, we include an icon with a word that describes each section so that students can become familiar with what each of the icons represents. In subsequent presentations, it only includes the icon that provides access to the section. Pressing the icon will take the presentation to the section it represents.

- **PDF Documents.** These documents include a copy of the practice exercises in the lesson, an additional practice section, activities to work on with a calculator, or homework. These documents can be printed so that students can work on them with pencil. The homework documents are exercises and problems that students can work on at home and allow them, through practice, to refine their newly acquired skills and concepts. Homework is optional.
- **Internet Links.** These links are a direct connection to the internet and can be accessed from the presentation. They include extra explanations, examples, applications, or



demonstrations that allow students to conceptually develop the skills and topics discussed.

### **Unit Documents**

- **Lesson 0.** It is not a lesson that provides content, but rather a file with elements inside the Unit: PDF documents, unit evaluations, and introductory video.
  - **PDF Documents.** Documents with laboratory activities for application and conceptual development, laboratory activities for the development of skills, activities using a calculator, extra exercises, and assessment activities.
  - **Evaluations.** This includes the evaluations to be used in the unit: the unit pretest and post-test. Lesson 00 of Unit I includes a diagnostic test for the entire course.
  - **Real Zone.** It consists of a short video that presents situations or elements in our immediate environment, in which we concretely visualize the use and application of concepts and skills to be studied in the Unit. In this video, we can introduce the topic to be studied, presenting the usefulness in daily life of what the student will learn.

## Unit Breakdown

Below, we present a breakdown of the content of the course with its units, lessons, and general objectives per unit, unit topic or title for each lesson, as well as the specific objectives and concepts per lesson.

### Unit 1. Elements of Geometry

This unit frames the study of geometry from its historical facet and how the ancient civilizations contributed to the construction of concepts and mathematical skills related to geometry. We also present the contributions of relevant historical characters who have been central figures in the development of geometry. Essential elements such as axioms, postulates, theorems, and the proof process, are a few of the topics developed in this unit.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

#### General Objectives

- Discuss the objective of studying geometry, the events, and relevant historical characters.
- Identify the basic elements of geometry using the correct nomenclature.

### Lesson 1. Historical Elements of Geometry

Code: C324G0SU01L01

#### Objectives

- State the objective of the study of geometry.
- Investigate and recognize the contributions of the Babylonian, Egyptian, and Greek civilizations to geometry.

#### Keywords

- Babylon
- Egypt
- Greece

## **Lesson 2. Great Mathematical Thinkers**

**Code:** C324G0SU01L02

### **Objectives**

- Identify the most relevant figures to the development of geometry.
- Investigate and recognize the contributions of different historical figures to the science of geometry.
- Establish a timeline related to thinkers and scholars of geometry throughout history.

### **Keywords**

- Thales of Miletus
- Pythagoras of Samos
- Euclid
- Archimedes of Syracuse
- Apollonius of Perga
- Hero of Alexandria

## **Lesson 3. Non Euclidian Geometry**

**Code:** C324G0SU01L03

### **Objectives**

- Explore the contributions of contemporary mathematicians to geometry.
- Contrast Euclidian and non-Euclidian geometry.
- Identify the contribution of different disciplines within geometry.

### **Keywords**

- non-Euclidian geometry
- Lobachevski
- Riemann

## **Lesson 4. Elements and General Ideas**

**Code:** C324G0SU01L04

### **Objectives**

- Define and recognize the inductive and deductive method.
- Identify the properties of equality.
- Define the characteristics of a geometrical body.

### **Keywords**

- inductive method
- deductive method
- properties of equality
- properties of inequalities
- geometrical bodies
- fallacies

## **Lesson 5. Terminology and General Concepts**

**Code:** C324G0SU01L05

### **Objectives**

- Define axiom and postulate.
- Define keywords such as, theorem, hypothesis, thesis, and corollary.
- Determine the hypothesis and thesis of a theorem.
- Determine the reciprocal theorem of a given theorem.

### **Keywords**

- axiom
- postulate
- theorem
- hypothesis
- thesis
- proof
- corollary
- reciprocal theorem

## **Unit 2. Points, Lines, and Planes**

This unit presents the elements or geometric bodies which are the basis of Euclidian geometry. The recognition of the notation and nomenclature of points, lines, and planes as essential to this unit. In this unit, we introduce the two column proof for theorems using the postulates and properties of equalities and real numbers. We also introduce concepts and skills related to the geometric bodies, especially, the segment, as well as operations with segments.

### **General Objectives**

- In a diagram, identify the elements of the line and its nomenclature.
- Carry out operations with segments.
- Identify postulates related to lines and segments.
- Prove the theorem of the inscribed and inscribing line.
- Prove theorems and solve problems related to the addition and subtraction of segment.

### **Lesson 1. Points and Lines**

**Code:** C324G0SU02L01

#### **Objectives**

- Identify points on a plane.
- Use correct nomenclature for points and lines.
- Represent postulates related to points and lines.
- Define and identify polygonal lines.
- Classify polygonal lines as convex or concave, open or closed.

#### **Keywords**

- notation
- types of lines
- open – closed
- concave – convex
- polygonal

### **Lesson 2. Lines**

**Code:** C324G0SU02L02

#### **Objectives**

- Identify and establish the correct nomenclature related to lines, segments, and rays.
- Discuss the postulates related to points, lines, and segments.
- Define the addition and subtraction of segments.
- Solve problems related to points, lines, and segments.
- Prove the law of monotony and the property of uniformity.
- Prove the theorem of the inscribed and inscribing line.

#### **Keywords**

- notation

- postulates
- ray
- segment (postulate)
- collinear points
- measurement and operations with segments
- theorem of the inscribed and inscribing line

### **Lesson 3. Plane and Surface**

**Code:** C324G0SU02L03

#### **Objectives**

- Identify and establish the correct nomenclature related to planes.
- Represent postulates related to points, lines, and planes.
- Identify points on a plane and the relationship between them.
- Identify coplanar points.

#### **Keywords**

- notation
- plane (postulates)
- coplanar
- half-plane

### **Lesson 4. Building Segments**

**Code:** C324G0SU02L04

#### **Objectives**

- Build the length of a segment, given its measurement.
- Use a segment transporter to build a congruent segment.
- Divide a segment into equal parts using a compass.

#### **Keywords**

- transportation of segments
- dividing segments into  $n$  parts

### **Unit 3. Angles**

This unit presents the definition and classification of angles. We present the sexagesimal system to measure angles and simultaneously discuss the relationship between angles, such as complementary angles, supplementary angles, and angles opposed by the vertex. We present different proofs in order to prove theorems related to angles. We establish the relationship between other systems of measurement of angles such as the radian. We present the transportation of a segment and the construction of congruent angles.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

#### **General Objectives**

- Define angle and related nomenclature.
- Classify angles by their measurements.
- Carry out operations with angles.
- Identify consecutive, adjacent, complementary, and supplementary angles.
- Prove the theorem of the angles opposed by the vertex.
- Prove theorems related to addition and subtraction of angles.

#### **Lesson 1. Angles**

**Code:** C324G0SU03L01

#### **Objectives**

- Define the concept of angles.
- Use the correct nomenclature to identify angles.
- Classify angles as acute, right, obtuse, or right, according to their measurements.
- Add and subtract angles in the sexagesimal system.
- Define the measurement of angles in radians.
- Convert measurements of angles from the sexagesimal system to radian and vice versa.

#### **Keywords**

- definitions
- notation/nomenclature
- measurement of angles
- sexagesimal system
- classification
- decimal system
- radian
- operations with angles

## **Lesson 2. Relationships Between Angles**

**Code:** C324G0SU03L02

### **Objectives**

- Define and compare adjacent and consecutive angles.
- Solve problems related with adjacent angles.
- Solve problems about complementary and supplementary angles.
- Determine the measurement of consecutive angles on a straight line and around a point.
- Prove the theorem of angles opposed by the vertex.
- Solve problems that involve angles opposed by the vertex, complementary angles, supplementary angles, consecutive angles on a straight line, and around a point.
- Prove theorems related to the addition and subtraction of angles, angles opposed by the vertex, complementary angles, and supplementary angles.
- Define the angle bisector.

### **Keywords**

- consecutive
- on a straight line
- around a point
- complementary
- supplementary
- opposed by the vertex
- bisector

## **Lesson 3. Building Angles**

**Code:** C324G0SU03L03

### **Objectives**

- Use a protractor to construct congruent angles.
- Use the compass to construct congruent angles.
- Trace the angle bisector.

### **Keywords**

- angle protractor
- protracting an angle
- angle bisector



#### **Unit 4. Perpendicular and Parallel Lines**

In this unit, we develop the concepts of parallelism and perpendicularity, and how they apply to geometric bodies, especially, lines and planes. We present the relationship between angles formed when a transversal line is traced to parallel lines and its related theorems. Additionally, we show the construction of parallel and perpendicular lines from different perspectives.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

#### **General Objectives**

- Define parallel line and perpendicular line.
- Find the measurement of angles between parallel lines intersected by a transversal.
- Prove theorems related to parallel and perpendicular lines.

#### **Lesson 1. Definition and Postulates About Perpendicular Lines**

**Code:** C324G0SU04L01

#### **Objectives**

- Define perpendicular lines.
- Represent postulates related to perpendicular lines.
- Identify and correctly use the nomenclature for perpendicular lines.
- Identify perpendicular planes.

#### **Keywords**

- theorem of a point outside a line
- perpendicular

#### **Lesson 2. Definition and Postulates About Parallel Lines**

**Code:** C324G0SU04L02

#### **Objectives**

- Define parallelism and parallel lines.
- Represent the postulates related to parallel lines.
- Correctly identify the nomenclature of parallel lines.
- Identify parallel planes.

#### **Keywords**

- parallel
- Euclid's postulates

### **Lesson 3. Lines Intersected by a Transversal**

**Code:** C324G0SU04L03

#### **Objectives**

- Identify a transversal line that intersects other lines.
- Represent postulates of parallel lines and Euclid's postulate.
- Name angles made by two lines intersected by a transversal.
- Prove the theorems related with alternate internal angles, alternate external angles, and same side internal and external angles between parallel lines.
- Solve problems related to corresponding angles, alternate internal and external angles between parallel lines.
- Apply the concepts of angles between parallel lines intersected by a transversal to solve problems of application.

#### **Keywords**

- transversal
- consecutive
- alternate internal angles
- alternate external angles
- same side internal angles
- same side external angles

### **Lesson 4. Relationships Between Angles With Parallel Sides**

**Code:** C324G0SU04L04

#### **Objectives**

- Prove the properties of angles that have parallel sides.
- Solve problems of application using the properties of angles that have parallel sides.

#### **Keywords**

- parallel
- perpendicular

### **Lesson 5. Constructs With Perpendicular and Parallel Lines**

**Code:** C324G0SU04L05

#### **Objectives**

- Construct a perpendicular line and a line through a point outside it.
- Construct a perpendicular line that contains point on a line.
- Trace a perpendicular line through one end of a segment.
- Trace a perpendicular line through the midpoint of a segment (perpendicular bisector).
- Construct a line that is parallel to another through an exterior point.

#### **Keywords**

- perpendicular
- exterior point
- interior point
- end of a segment
- parallel
- perpendicular bisector

## **Unit 5. Triangles**

The purpose of this unit is to develop a broad overview of the triangle and its characteristics. We develop advanced concepts such as notable points and lines in the triangle (altitude, medians, perpendicular bisectors, and bisectors), and their notable points. We develop theorems related to the sum of the interior angles and the measurement of an exterior angle. The lesson about constructs provides an advanced overview of the importance of the orthocenter, barycenter, circumcenter, and incenter, and about how they can be applied in real life.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Define the triangle as an area limited by three lines that intersect two to two.
- Identify the elements of a triangle.
- Classify triangles by the measurement of their sides and angles.
- Identify the notable points and lines in a triangle.
- Prove the theorem of the sum of the interior angles in a triangle.
- Prove the theorem related to the measurement of an exterior angle of a triangle.
- Establish the relationship between the measurement of a side and its opposing angle.
- Solve application problems related to theorems of a triangle.
- Determine and identify the properties of an equilateral triangle.
- Determine and identify the properties of an isosceles triangle.
- Solve problems related to the properties of equilateral and isosceles triangles.

### **Lesson 1. Triangles**

**Code:** C324G0SU05L01

#### **Objectives**

- Define the concept of a triangle as an area limited by three lines that intersect two to two.
- Identify the elements of the triangle.
- Classify triangles by the measurement of their sides and angles.
- Establish the relationship between the measurement of a side and its opposing angle.

#### **Keywords**

- triangle
- equilateral
- isosceles
- scalene
- acute triangle

- right triangle
- obtuse triangle
- relationship between angles and sides

## **Lesson 2. Angles of the Triangle**

**Code:** C324G0SU05L02

### **Objectives**

- Prove the theorem of the sum of the interior angles of a triangle.
- Prove the theorem related to the measurement of the exterior angle of a triangle.
- Solve application problems related to the theorems of a triangle.

### **Keywords**

- theorem of the sum of the interior angles of a triangle
- theorem of the measurement of the exterior angles of a triangle
- sum of the exterior angles of a triangle

## **Lesson 3. Notable Points and Lines**

**Code:** C324G0SU05L03

### **Objectives**

- Identify and trace the notable points and lines in a triangle.
- Establish the relationship between the barycenter and the distance to the midpoint and the vertex.
- Establish the relationship between the circumcenter and the distance to the vertexes.
- Establish the relationship between the incenter and the distance to the sides of the triangle.
- Identify the relationship between the notable points and the Euler line.

### **Keywords**

- altitude
- orthocenter
- medians
- barycenter
- perpendicular bisector
- circumcenter
- bisector

## **Lesson 4. Properties of the Equilateral and Isosceles Triangles**

**Code:** C324G0SU05L04

### **Objectives**

- Determine and identify the properties of equilateral triangles.
- Determine and identify the properties of the isosceles triangles.
- Solve problems related to the properties of equilateral and isosceles triangles.

### **Keywords**

- Relationship between altitudes, median, perpendicular bisector, and bisector in an equilateral triangle.
- Relationship between altitudes, median, perpendicular bisector, and bisector in an isosceles triangle.

### **Lesson 5. Constructs With Notable Points and Lines**

**Code:** C324G0SU05L05

#### **Objectives**

- Construct the altitudes of a triangle and identify the orthocenter.
- Construct the medians of a triangle and identify the barycenter.
- Construct the perpendicular bisectors of a triangle and identify the circumcenter.
- Construct the bisectors of a triangle and identify the incenter.
- Use geometric constructs to deduce properties of right triangles, obtuse triangles, equilateral triangles, and isosceles triangles.
- Draw the Euler line.

#### **Keywords**

- altitude
- orthocenter
- median
- barycenter
- perpendicular bisector
- circumcenter
- bisector
- incenter
- Euler line

## **Unit 6. Congruence and Similarity in Triangles**

The purpose of this unit is to compare the characteristics that are shared or different between two figures, in this case, congruent and similar triangles. We discuss the concept of ratio and proportion as a basis for proving advanced theorems in geometry, such as the Pythagorean Theorem. The skills and concepts developed in this unit will provide the student with the opportunity to acquire the basics for other advanced courses such as Trigonometry and Precalculus.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Determine the congruence of two triangles using the congruence criteria *SSS*, *SAS*, and *ASA*.
- Define and apply the concept of ratio.
- Establish the proportion between two ratios.
- Solve proportions using corresponding properties.
- Solve problems related to proportionality of segments.
- Determine whether two triangles are similar by applying the *AA*, *ASA*, and *SSS* criteria.
- Demonstrate the theorem of Thales of Miletus and its application in problem solving.
- Apply the theorem relative to the parallel segment to a side of the triangle.

### **Lesson 1. Congruence of Triangles**

**Code:** C324G0SU06L01

#### **Objectives**

- Determine the congruence of two triangles using the congruence criteria *SSS*, *SAS*, and *ASA*.
- Determine the congruence of right triangles.
- Solve problems related to the congruence of triangles.

#### **Keywords**

- *SSS* criteria
- *ASA* criteria
- *SAS* criteria
- *AA* criteria
- congruence of right triangles

## **Lesson 2. Ratios and Proportions**

**Code:** C324G0SU06L02

### **Objectives**

- Define ratio and proportion.
- Identify the components of a proportions.
- Identify and demonstrate the properties of proportionalities.
- Solve proportions using cross multiplication.
- Solve problems that relate the properties of proportionality.

### **Keywords**

- ratio
- proportion
- properties of proportions
- fourth proportional of three quantities
- third proportional to two quantities
- proportional mean to two quantities

## **Lesson 3. Proportional Segments**

**Code:** C324G0SU06L03

### **Objectives**

- Demonstrate the theorem of Thales of Miletus regarding parallels cut by two transversals.
- Apply Thales' theorem for solving triangles.

### **Keywords**

- proportional segments
- transversals
- Thales' theorem

## **Lesson 4. Theorems Related to a Segment Parallel to One Side of a Triangle**

**Code:** C324G0SU06L04

### **Objectives**

- Apply the theorem relative two the midpoints of the sides of a triangle.
- Demonstrate the theorem related to an angle bisector and determined segments on the opposing side.

### **Keywords**

- segment parallel to one side of a triangle
- angle bisector theorem

## **Lesson 5. Similarity of Triangles**

**Code:** C324G0SU06L05

### **Objectives**

- Find the similarity between triangles using the *AA*, *ASA*, and *SSS* criteria.
- Solve application problems, using the similarity of triangles and the property of proportions.

### **Keywords**

- similarity
- criteria of similarity



## **Unit 7. Metric Relationships Between the Sides of a Triangle**

In the study of geometry and other related topics, the Pythagorean Theorem occupies a very special place. This unit begins with the concept of segment projection and continues with the theorem related to the altitude of the hypotenuse, with the objective of establishing the basis for proving the Pythagorean Theorem. The application of this theorem will allow students to develop the necessary skills to face topics in advanced courses such as the formula for distance, trigonometry and the solution of triangles.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Represent the projection of a point or of a segment on a line.
- Prove a theorem related to the altitude of the hypotenuse and its corollaries.
- Demonstrate the Pythagorean Theorem.
- Apply the Pythagorean Theorem to solve triangles.
- Prove the generalization of the Pythagorean Theorem to classify triangles.

### **Lesson 1. Projections**

**Code:** C324G0SU07L01

#### **Objectives**

- Determine the projections of a point and a segment on a line.

#### **Keywords**

- projection

### **Lesson 2. Theorem Relative to the Altitude of the Hypotenuse of a Triangle**

**Code:** C324G0SU07L02

#### **Objectives**

- Demonstrate the theorem related to the altitude of the hypotenuse in a right triangle.

#### **Keywords**

- projection of a hypotenuse
- altitude corresponding to the hypotenuse

### **Lesson 3. Pythagorean Theorem**

**Code:** C324G0SU07L03

#### **Objectives**

- Prove the Pythagorean Theorem.
- Solve problems involving the use of the Pythagorean Theorem.

#### **Keywords**

- hypotenuse
- leg
- proofs

### **Lesson 4. Generalization of the Pythagorean Theorem**

**Code:** C324G0SU07L04

#### **Objectives**

- Develop the generalization of the Pythagorean Theorem.
- Prove the theorem related to the value of the square of the opposing side to an obtuse angle in a triangle.
- Classify a triangle using a Pythagorean Theorem.

#### **Keywords**

- Pythagorean Theorem
- acute triangle
- right triangle
- obtuse triangle

## **Unit 8. Polygons**

The study of the triangle has allowed us to establish the basis to find the properties of theorems of other polygons. In this unit, we present the analysis of the characteristics of regular and irregular polygons, and theorems related to the sum of their interior and exterior angles. Also, we establish the formulas to determine the number of diagonal lines that can be traced in any polygon. As part of the study of polygons, we present the study of quadrilaterals, paying mind to their classification and properties.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Classify polygons by the number of their sides.
- Determine whether a polygon is regular or irregular, convex or concave.
- Find the sum of the angles of any polygon.
- Find the measurement of an interior angle in a regular polygon.
- Prove the theorem of the sum of the interior angles of any polygon.
- Find the measurement of the exterior angle of a regular polygon.
- Classify quadrilaterals according to their characteristics and properties.
- Solve problems related to quadrilaterals.
- Apply transformations to polygons on the Cartesian plane.

### **Lesson 1. Definition and Classification of Polygons**

**Code:** C324G0SU08L01

#### **Objectives**

- Classify polygons by the number of their sides.
- Identify and classify polygons as concave or convex.
- Identify and classify polygons as regular or irregular.

#### **Keywords**

- concave and convex
- regular and irregular

### **Lesson 2. Angles and Diagonals in Polygons**

**Code:** C324G0SU08L02

#### **Objectives**

- Find the sum of the interior angles and exterior angles of any polygon.
- Find the measurement of an interior and exterior angle in a regular polygon.
- Trace diagonals in any polygon.
- Find the number of diagonals traced from the vertex of a polygon.
- Find the total of diagonals that can be traced in a polygon.

**Keywords**

- diagonal
- interior angle
- exterior angle

**Lesson 3. Quadrilaterals****Code:** C324G0SU08L03**Objectives**

- Classify quadrilaterals according to their characteristics and properties.
- Identify the properties and characteristics of each quadrilateral.
- Use the properties of quadrilaterals to solve problems.

**Keywords**

- rhombus
- parallelogram
- trapezium
- kites
- trapezoids

**Lesson 4. Symmetry and Transformation****Code:** C324G0SU08L04**Objectives**

- Define the different kinds of transformations in figures.
- Carry out transformations on the Cartesian plane.
- Identify whether two shapes are congruent or similar.

**Keywords**

- reflection
- axis of symmetry
- translations
- rotations
- dilation
- similarity

## **Unit 9. Circumferences and Circles**

This unit presents the geometric bodies of circumference and circles, and the elements that make it up. We analyze the relationship between the angles that intersect the arcs of a circumference and their relationship with the measurement in degrees.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Define circumference and circle.
- Establish the difference between circumference and circle.
- Identify the elements related to circumference and circles.
- Classify two circumferences according to the distance of its centers.
- Find the measurement of the angles that intersect a circumference according to its length.
- Construct a tangent line to a circumference.
- Construct interior and exterior tangents between two cosecant, tangent, or exterior circumference.

### **Lesson 1. Definition and Elements of the Circumference and the Circle**

**Code:** C324G0SU09L01

#### **Objectives**

- Define circumference and circle.
- Identify the parts of the circle and the circumference.
- Prove theorems related to circumference.
- Prove theorems related to tangents of a circumference.

#### **Keywords**

- circumference
- circle
- radius
- chord
- diameter
- arc
- secant

## **Lesson 2. Relationship Between the Centers of Two Circumferences**

**Code:** C324G0SU09L02

### **Objectives**

- Classify the relationship between two circumferences according to the distance between the centers.
- Prove theorems related to distances between the centers of two circumferences.

### **Keywords**

- interiors
- exteriors
- exterior tangent circumference
- interior tangent circumference
- secants

## **Lesson 3. Angles in Circumferences**

**Code:** C324G0SU09L03

### **Objectives**

- Classify angles related to the circumference as central, interior, inscribed, semi-inscribed, or exterior.
- Establish the relationship between angles and the measurement of the arc intersected by it.
- Determine the measurements of angles related to circumferences.

### **Keywords**

- central
- interior
- inscribed
- semi-inscribed
- exterior

## **Lesson 4. Constructing Tangents**

**Code:** C324G0SU09L04

### **Objectives**

- Construct a line tangent to a circumference.
- Construct interior tangents between two exterior circumferences.
- Construct exterior tangents to two secant, tangent, or exterior circumferences.

### **Keywords**

- tangent to the circumference by a point
- exterior lines tangent to two circumferences
- interior lines tangent to two circumferences

## **Unit 10. Perimeter and Area**

In this unit, we present two basic concepts for geometry: the perimeter and area of plane figures. We present the basic formulas to determine the perimeter and area of the triangle, quadrilaterals, and other regular polygons. We also present the formula to determine the length of a circumference and the area of a circle.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Define perimeter and area.
- Find the perimeter and area of triangles and quadrilaterals.
- Find the perimeter and area of regular polygons.
- Calculate the circumference and area of a circle.
- Find the perimeter and area of plane compound shapes.

### **Lesson 1. Perimeter and Area of Triangles**

**Code:** C324G0SU10L01

#### **Objectives**

- Find the area of triangles using the formula for base and height.
- Use Hero's formula to find the area of a triangle given the lengths of its sides.

#### **Keywords**

- half the base by height
- Hero's formula

### **Lesson 2. Perimeter and Area of the Quadrilaterals**

**Code:** C324G0SU10L02

#### **Objectives**

- Find the area of a quadrilateral using the formula of base and height.
- Use the length of diagonals to determine the area of quadrilaterals.
- Find the area of a trapezoid using the midsegment and height.

#### **Keywords**

- quadrilateral
- base
- midsegment
- height
- diagonal

### **Lesson 3. Perimeter and Area of the Polygons**

**Code:** C324G0SU10L03

#### **Objectives**

- Find the perimeter of a regular polygon.
- Find the area of a regular polygon using the apothem.

#### **Keywords**

- base
- height
- diagonal
- apothem

### **Lesson 4. Circumference and Circle**

**Code:** C324G0SU10L04

#### **Objectives**

- Find the length of the circumference.
- Find the length for the arc of a circumference.
- Find the area of a circle.
- Find the area of a circular section.
- Find the area of a trapezium.
- Find the area of s circular segment.

#### **Keywords**

- circular section
- annulus
- circular trapezoid
- circular segment

### **Lesson 5. Compound Areas and Regions**

**Code:** C324G0SU10L04

#### **Objectives**

- Find the area of plane compound figures.

#### **Keywords**

- subtraction of regions
- addition of regions



## **Unit 11. Geometry in Space**

This unit presents three-dimensional figures and their classification. Within the set of three-dimensional bodies, polyhedrons stand apart; among them parallelepiped, prisms, pyramids, and Platonic solids. We also present the study of round bodies such as cylinders, cones, and spheres, and their characteristics.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Classify three-dimensional figures as polyhedrons or round bodies.
- Identify the characteristics of polyhedral.
- Identify the characteristics of round bodies.

### **Lesson 1. Three-Dimensional Shapes Polyhedrons**

**Code:** C324G0SU11L01

#### **Objectives**

- Identify solid shapes as round bodies.
- Draw the transverse view of polyhedrons.
- Construct nets of Platonic solids.
- Identify polyhedrons in the environment.

#### **Keywords**

- prisms
- pyramids
- Platonic solids

### **Lesson 2. Three-Dimensional Figures: Round Bodies**

**Code:** C324G0SU11L02

#### **Objectives**

- Identify solid shapes as round bodies.
- Draw the transverse view of round bodies such as cones, cylinders, and spheres.
- Identify round bodies around them.

#### **Keywords**

- cone
- cylinder
- sphere

## **Unit 12. Surface Area and Volume**

In this unit we will present the concept of surface area and volume of three-dimensional figures such as polyhedrons and round figures. We discuss the process for determining the surface area and volume of figures such as pyramids, and prisms such as cubes and parallelepipeds. Also, we present the calculation of surface area and volume of round figures such as the cylinder, cone, and sphere. We also present the process to find the surface area and volume of compound figures.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

### **General Objectives**

- Define surface area and volume.
- Find the area of three-dimensional shapes such as polyhedrons.
- Find the surface area and volume of round bodies.
- Calculate the surface area of compound three-dimensional shapes.

### **Lesson 1. Surface Area and Volume of Polyhedrons**

**Code:** C324G0SU12L01

#### **Objectives**

- Find the surface area of prisms and pyramids.
- Find the volume of prisms and pyramids.

#### **Keywords**

- cubes
- prisms
- pyramids

### **Lesson 2. Surface Area and Volume of Round Bodies**

**Code:** C324G0SU12L02

#### **Objectives**

- Find the surface area of cylinders, cones, and spheres.
- Find the volume of cylinders, cones, and spheres.

#### **Keywords**

- cylinder
- cone
- sphere

### **Lesson 3. Surface Area and Volume of Compound Figures**

**Code:** C324G0SU12L03

#### **Objectives**

- Find the surface area of compound figures.
- Find the volume of compound figures.
- Compare the volume of prismatic and round figures.

#### **Keywords**

- volume
- surface area

### **Unit 13. Elements of Trigonometry**

The purpose of this unit is to introduce the elemental concepts of trigonometry based on topics studied in the Geometry course. Part of the mathematical content discussed in this unit is: the analysis of the right triangle, the Pythagorean theorem; the ratio between the segments of a triangle, the trigonometric ratios; trigonometric values, and the trigonometric values of  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  angles, and the relationship between its collaterals. These topics will allow students to develop the necessary skills and concepts to begin the high school Trigonometry course.

At the end of this unit, the student will have completed the following objectives as developed in the different lessons.

#### **General Objectives**

- Identify the general use of trigonometric functions.
- Define the trigonometric functions of an acute angle in a right triangle.
- Solve right triangles using trigonometric functions.
- Find the trigonometric values of 30, 60, and 45 degree angles.
- Find the trigonometric values for angles formed in any quadrant of the Cartesian plane.
- Identify positive angles, negative angles, and angles greater than 360 degrees.
- Find the trigonometric values for angles formed on the Cartesian plane.
- Simplify angles and determine its equivalent between 0 and 360 degrees.
- Find the trigonometric values for any angle equivalent to 30, 45, and 90 degrees.
- Transform the measurement of angles from the sexagesimal system to radians and vice versa.
- Solve right triangles using the Pythagorean Theorem and trigonometric ratios.

#### **Lesson 1. Elements of Trigonometry**

**Code:** C324G0SU13L01

##### **Objectives**

- State the importance of trigonometry within the historical framework of humankind.
- Define and investigate the origins of trigonometry.
- Review concepts and theorems related to the right triangle.

##### **Keywords**

- historical aspects
- right triangle
- hypotenuse
- leg
- similar triangles

## Lesson 2. Trigonometric Ratios

Code: C324G0SU13L02

### Objectives

- Find ratios between the sides of a right triangle.
- Define trigonometric ratios of an angle in a right triangle.
- Find the trigonometric ratios of an angle in a right triangle using trigonometric functions and the Pythagorean Theorem.

### Keywords

- sine
- cosine
- tangent

## Lesson 3. Trigonometric Ratios for $30^\circ$ , $45^\circ$ , and $60^\circ$

Code: C324G0SU13L03

### Objectives

- Prove the trigonometric values for 30 and 60 degree angles using an equilateral triangle and the height corresponding to one of the vertexes.
- Prove trigonometric values of a 45 degree angles using an isosceles triangle.
- Solve problems related to 30, 45, 60 degree angles.
- Find the value of trigonometric expressions that contain 30, 45, 60 degree angles.

### Keywords

- $30^\circ$  and  $60^\circ$  trigonometric values
- $45^\circ$  trigonometric values

## Lesson 4. Trigonometric Values for Angles Greater Than $90^\circ$

Code: C324G0SU13L04

### Objectives

- Determine the trigonometric functions of an angle made up of a terminal side that contains a point on the Cartesian plane and  $x$  axis.
- Define the sign corresponding to trigonometric functions according the quadrant on the Cartesian plane.
- Determine the trigonometric values for quadrant angles formed on the axes, as 90, 180, 270, and 360 degrees.
- Determine the value of trigonometric expressions that contain 90, 180, 270, and 360 degree angles.
- Find a reference angle for any angle on the Cartesian plane.
- Find the equivalent positive angle to a negative angle.
- Convert an angle to its positive equivalent less than 90 degrees.
- Find the trigonometric values of negative angles.
- Simplify angles greater than 360 degrees.
- Find trigonometric values for angles greater than 360 degrees.
- Express angles from the sexagesimal system to radians.

**Keywords**

- Find trigonometric values of angles in radians.
- quadrants
- quadrantile angle
- reference angle
- coterminal side
- radian

**Lesson 5. Solving Right Triangle**

**Code:** C324G0SU13L05

**Objectives**

- Solve right triangles using the Pythagorean Theorem and trigonometric functions in right triangles.
- Apply the inverse functions of trigonometric ratios to determine the value of the angles of a right triangle.
- Solve application problems using trigonometric functions in right triangles.

**Keywords**

- inverse sine
- inverse cosine
- inverse tangent
- angle elevation and depression