DREYFOUS

Thematic Guide

CALCULUS

DREYFOUS

TABLE OF CONTENTS

Course Description

The Calculus course of EduSystem has as a fundamental objective of developing high-level mathematical skills in students, as well as to raise awareness of the importance of studying calculus for resolving problems and situations in daily life. Through the developed content and the used strategies and methods, the students will gain a deep insight of the concepts, as well as the technical skills needed to study university courses or to prepare for advanced-level tests. The approach to multiple representations is used through all the course to make easier the visualization of concepts. The way in which the subjects and examples are presented, how the applications are used, as well as how the mathematical skills are developed, allows students to visualize, understand, and value the use of math in everyday life.

The content of the course aligns with the curriculums of most renowned universities in Puerto Rico and to a great extent, with the curricular sequence of the advanced program College Board (AP). Part of the content follows the *Puerto Rico Core Standards* of the Puerto Rico Department of Education (2014) and the United States *Common Core State Standards.* They also emphasize the standards of algebra and functions, also integrating areas of numeration and operation, geometry, data analysis, and probability. The standard of functions is worked with special care, and particular attention was given to the graph representation of polynomial and transcendental functions, such as trigonometric, exponential, and logarithmic ones.

The content of the course considers the three main subjects of any calculus course: limits, derivatives, and integrals (antiderivatives). The lessons related to limits consider the following topics: concept of function limits, trigonometric limits, limits related to infinity, formal definition of limits, and continuity of a function. Lessons related to derivatives study the following topics: the slope of the tangent line, the derivative, basic rules of differentiation, derivative of transcendental functions, the tangent line, the tangent and normal line, the differentiation of composite, inverse, and implicit functions, the chain rule, differentiation of inverse functions, application of the first and second derivatives, concavity and inflection points, graph analysis, and applications of the derivative.

The lessons related to antiderivatives and integrals contemplate the following topics: concept of derivatives, substitution method and change of variables, differentiable equations and their applications, antiderivatives of transcendental functions, integrals of inverse functions, fundamental theorem of calculus, summation notation, definite

integrals, area of regions, area and volumes of revolution, arc length and surface area, and applications of work, force, and power.

The first unit is designed to offer a review and reference content of fundamental subjects in the precalculus course. This unit contemplates the following subjects: real numbers, equations and inequalities, functions, and trigonometric, exponential, and logarithmic functions.

We also emphasize the mathematical processes of problem solving to promote the integration of the subjects and encourage the communication and acquisition of concepts, and the domain of the standards with efficiency.

The objectives designed by lesson consider the needed skills and concepts so the student can carry out the connections between the different math standards establishes by the Puerto Rico Department of Education (DEPR), the *Common Core State Standards,* and the recommendations by the *National Council of Teachers of Mathematics (NCTM)* related to the math teaching strategies.

The teaching focus is centered on conceptual understanding, critical thinking skill development, and solution of math problems as a means of the integral formation of the students. Specifically:

- Understanding the problems as developing the capacity to resolve them with confidence.
- Reasoning in a concrete and semi concrete manner until reaching the quantitative abstraction.
- Building and defending viable arguments, as well as understanding and analyzing the arguments and reasonings of others.
- Using math to resolve daily problems.
- Using the appropriate and needed tools (including technology) to resolve problems in different contexts.
- Being accurate in reasoning and in discussions with others.
- Distinguishing and using patterns or structures.
- Identifying and expressing regularity in repeated reasonings.

We also expect that the students communicate appropriately by using math terminology and that they incorporate technology accordingly in their learning process.

Course Structure

The course Calculus DA consists of nine units carefully subdivided into different lessons. The quantity of lessons per unit depends on the extent and depth with which the subjects are discussed and developed. Each lesson has an interactive presentation divided into sections that explain and show the contents of the subject to study. Each presentation includes conceptual definitions, concrete examples, explanations, multiple representations, practice exercises, applications of the concepts, and skills to use in daily life.

Additionally, the lessons include practice exercises, quizzes, homework, self-assessment exercises, and a descriptive log with detailed information for the teacher, as well as a variety of Internet links, among other resources. The activities are varied and flexible, with the purpose of satisfying the particular needs and interests of each student. The practice and self-assessment activities are meant to make the students think about their domain of the content and to take control of their learning gradually. The teachers, as an integral and essential part of the process, have the responsibility of stimulating, guiding, and evaluating periodically the learning accomplished by each student.

The units are composed of the following parts:

Lessons

Each unit is composed of different lessons, divided by subjects, macro concepts and skills. Additionally, each lesson has five fundamental elements: presentations or course content, digital documents (PDF), Internet links, self-assessment, and a descriptive log.

- **Presentation (*Lesson Content*).** Each presentation has a detailed explanation of the lesson's concepts and skills, according to the established objectives. In addition, it is composed of the following elements that contribute systematically with the development of the desired learning on students:
  - **Examples.** In each lesson, when developing skills, there are examples that explain, step by step, the solution to an exercise or problem, so that students can review the presented concepts and skills.

  - **Practice.** Includes a series of exercises carefully chosen to have students practice the discussed skills and concepts. It has the purpose of verifying periodically what the students have learned before continuing with other subjects and skills. Does not include processes or explanations, only solutions to the exercises.

  - **Answer.** Used to hide the solution for an exercise or problem while students try answering it. Once you click on this icon, the solution or answer to the exercise will appear.

  - **Process.** Label where the steps or algorithm to follow when solving an exercise or problem appear.

  - **Proof/Steps.** Presents formal demonstrations of the derivation of important formulas and algorithms.

  - **Calculator.** Includes the explanation of the processes in the use and management of the calculator to solve exercises in the lesson. It also connects the students with the virtual graphing calculator.

- **Animation.** Access to the explanations, processes, or graphs that visually show the concepts and skills discussed in the section. Helps with the development and conceptual understanding of students.

- **Definition.** Includes formal definitions of mathematical concepts and processes mentioned or discussed.

- **Biography.** Includes a short biography of the mathematician or scientist attributed to the development of the definition, formula, process, or demonstrations used in the lesson.

- **Note.** Under this icon we specify common mistakes, or we emphasize details that should be remembered.

- **Did You Know…** This is a section that presents an explanation or situation that connects aspects of daily life with the discussed skills and mathematical concepts. In some cases, this section shows the link between the development of logical thinking in human beings with certain skills and mathematical processes.

- **Tabs.** Located at the right side of the presentation and can be burgundy or blue. They unfold to the left and include flow charts, biographies, notes, pictures, explanations, suggestions, reminders, "Did You Know…?" or necessary previous knowledge.

- **Incorrect.** Indicates when the students have chosen an incorrect answer in the included practice exercises.

- **Correct.** Indicates that the correct answer has been chosen in a practice exercise or problem.

- **Picture or Image.** Connects a particular explanation with a picture or image that probably access to the Internet.

- **Video.** Access a short video that links the mathematical content with daily life.

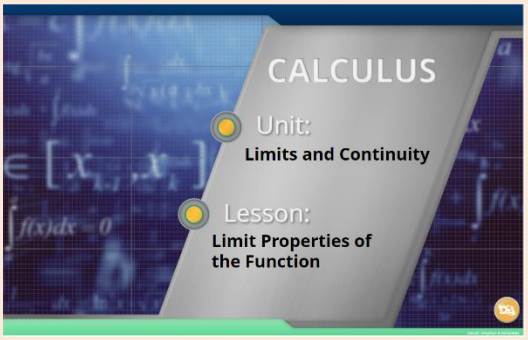
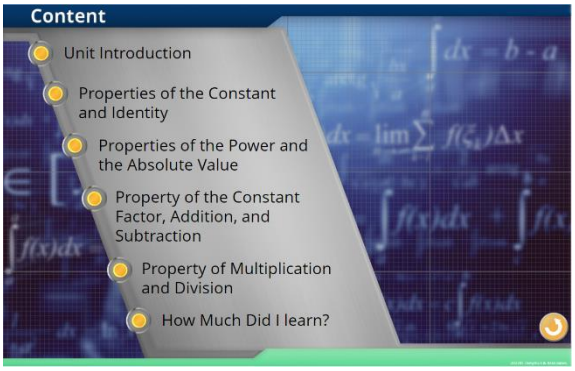
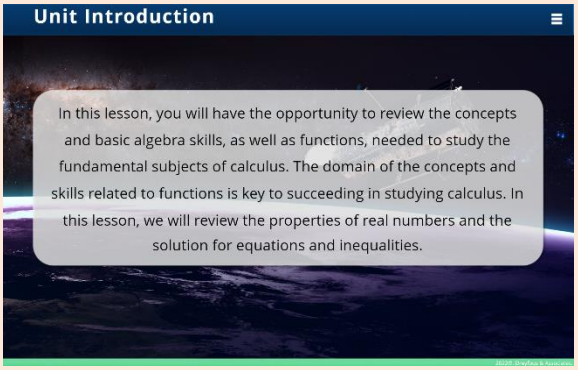
- **Internet.** A direct link to a site or Internet portal closely related to the subject.

Each of the sections included in the presentation is connected to particular icon that identifies it with the explanation. In the initial presentations of the course, we include the icon with the word that describes the section, so that students can familiarize themselves with the representation of the icons. In subsequent presentations, we only include the icon that accesses the section. Clicking the icon will instantly take the presentation to the specific section that it represents.

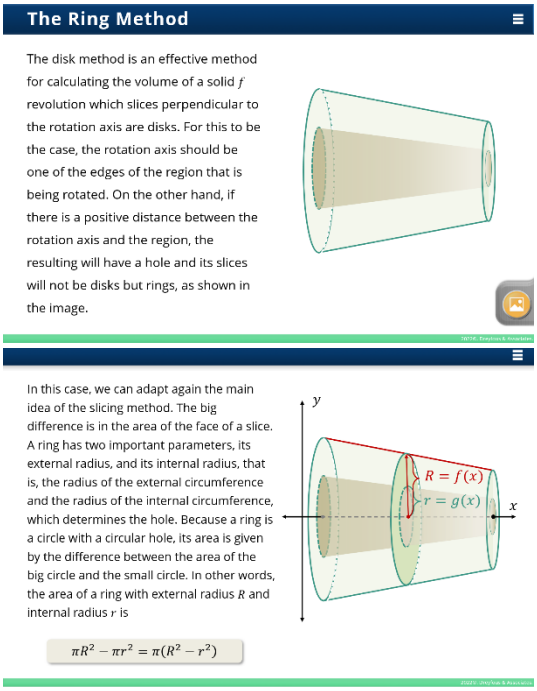
- **PDF Documents.** These documents include a copy of the practice exercises in the lesson, an additional practice section, activities to work with the calculator, or homework. These documents can be printed so that students can work on them by hand. The homework is composed of exercises and problems that students can work on at home and allow them, through practice, to refine the learned skills and concepts. Homework is optional.

- **Links.** These links are a direct connection to the Internet and can be accessed straight from the presentation. These include additional explanations, examples, applications, or demonstrations that allow the conceptual development of students in the skills and discussed topics.
- **Self-Assessment.** Consists of practice tests that students answer to track what they had learned before taking the formal evaluation tests of the unit, offered by the teacher.
- **Descriptive Log.** Is a detailed lesson plan. This includes the specific objectives of the lessons, standard and expectations, teaching strategies and resources, keywords, links, references, among others. Only the teacher will have access to the descriptive log of each lesson.

Curricular Components

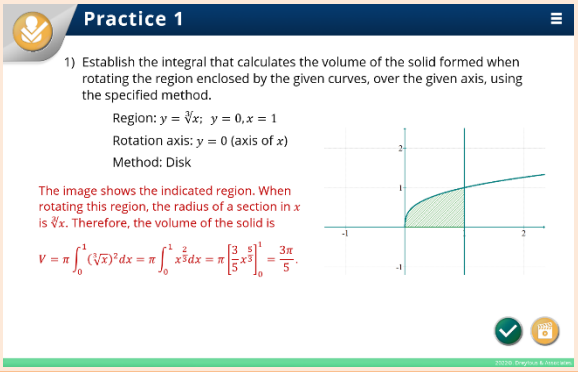
Lesson content	
Title Page 	Is the presentation letter of the course and lesson. Identifies the course, unit, and lesson. Contains: ○ Course title ○ Unit and lesson title ○ Image ○ Credits
Directory 	Presents the sections and topics of the lesson. Each button has a link to the section it represents.
Introduction 	This section can be found in all lessons.

Subjects (content)



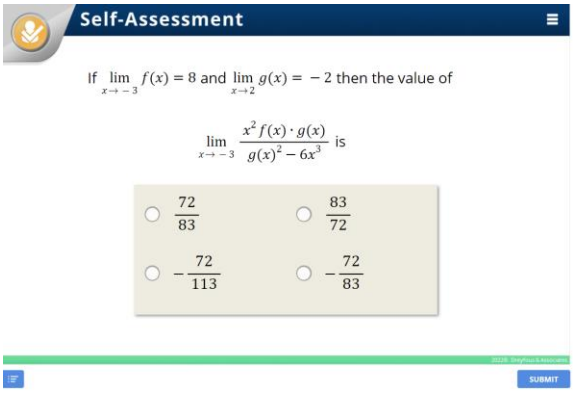
Content development through definitions, explanations, examples, and demonstrations.

How Much Did I Learn?



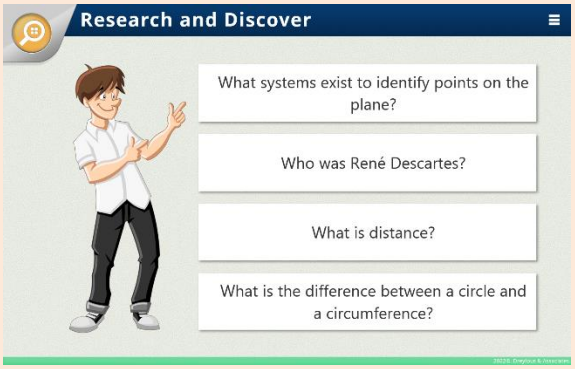
Short and objective exercises aimed at the evaluation and application of knowledge, located at the end of the subject. Contains the answers.

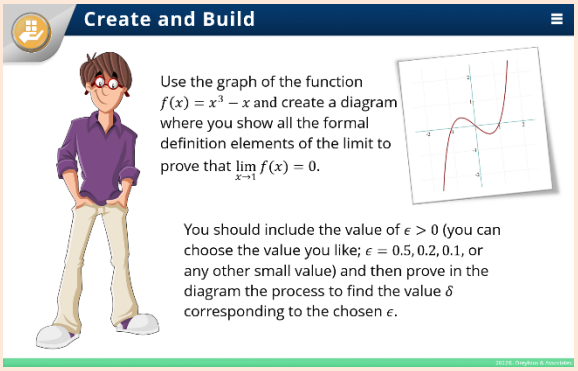
Self-Assessment

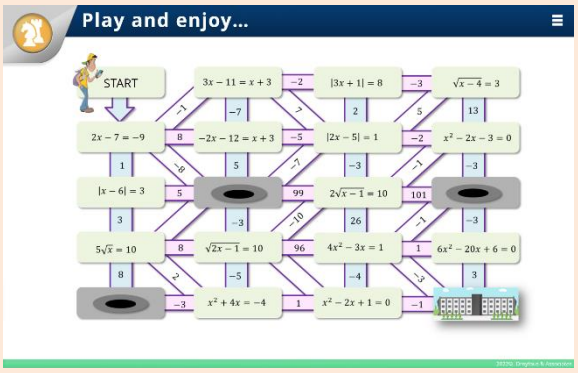


Consists of practice tests that students answer to track what they had learned before taking the formal evaluation tests of the unit, offered by the teacher.

Special Sections







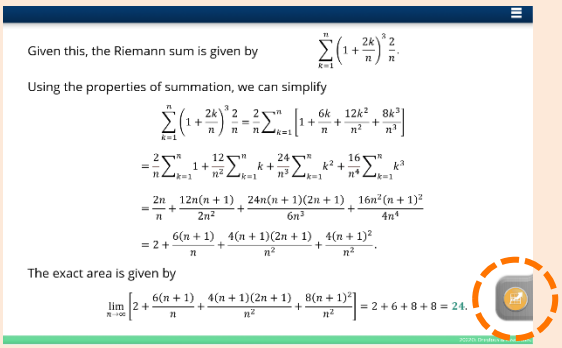
They broaden the learning experiences of students. One or more may appear in the lesson. They will be included as relevant. These lessons are:

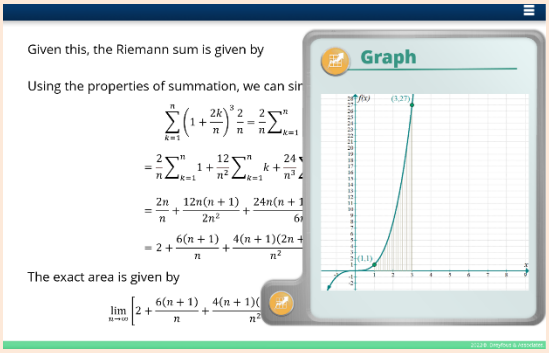
Research and Discover: Research of additional topics or integration with other classes.

Create and Build: Applications of what has been learned and creation of projects.

Play and enjoy: Integration of playful strategies for concepts and skills development.

The Tab





When clicking the gray tab at the top or bottom right of some of the *Lesson Content* slides, the students will see some important notes to reinforce or clarify the content, such as formulas or previous definitions.

Button Directory

Navigation

	Close		Credits
	Answer		Return

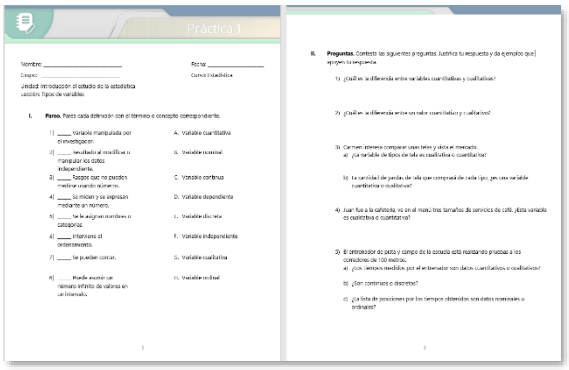
General

	Animation		Practice
	Link		Reason

	Definition		Review
	Biography		Steps
	Connect What You Have Learned		Graph
	Image		Calculator
	Note		Example
	Diagram		Self-Assessment
	Question		Process
	Video		Zoom
	Did You Know…?		Text
	Challenge Your Mind		
Special Sections			
	Research and Discover		Create and Build
	Think and Play		

Work Documents

Practice and Activities


Práctica 1

Nombre: _____ Fecha: _____

Grupo: _____ Curso: Estadística

Unidad: Introducción al estudio de la estadística

Lección: Tipos de variables

I. **Pares.** Para cada definición con el término o concepto correspondiente.

1) ____ Variable manipulada por el investigador.
2) ____ Resultado al modificar o manipular los datos independiente.
3) ____ Valores que no pueden medirse usando números.
4) ____ Se miden y se expresan mediante un número.
5) ____ Se le asignan nombres o categorías.
6) ____ Intervienen el ordenamiento.
7) ____ Se pueden contar.
8) ____ Puede asumir un número infinito de valores en un intervalo.

A. Variable cuantitativa
B. Variable nominal
C. Variable continua
D. Variable dependiente
E. Variable discreta
F. Variable independiente
G. Variable cualitativa
H. Variable ordinal

II. **Preguntas.** Contesta las siguientes preguntas. Justifica tu respuesta y da ejemplos que apoyen la respuesta.

1) ¿Cuál es la diferencia entre variables cuantitativas y cualitativas?

2) ¿Cuál es la diferencia entre una valor cuantitativo y cualitativo?

3) Carmen intenta comparar usos bélicos y civiles en mercado.
a) ¿La variable de tipos de telas es cualitativa o cuantitativa?

b) La cantidad de partes de tela que compraré de cada tipo, ¿es una variable cuantitativa o cualitativa?

4) Juan fue a la cafetería, vio en el menú los tamaños de servicios de café. ¿Esta variable es cualitativa o cuantitativa?

5) El entrenador de pista y campo de la escuela está realizando pruebas a los corredores de 100 metros.
a) ¿Los tiempos medidos por el entrenador son datos cuantitativos o cualitativos?

b) ¿Son continuos o discretos?

c) ¿La lista de posiciones por los tiempos obtenidos son datos nominales o ordinales?

They can be found in all lessons and provide the key for the teacher.

- Practice 1 and 2

Units and Lessons Breakdowns

Unit 1. Functions

When completing these lessons, students will have completed the objectives found in the following lessons:

Lesson 1. Real Numbers, Equations, and Inequalities

Code: C351G0SU01L01

Objectives

When completing this lesson, the student will:

- Classify real numbers and identify their properties.
- Solve equations of first degree, quadratic, rational, irrational, and of absolute value.
- Solve inequalities, write the solution sets in interval notations, and sketch graphs.

Subjects

- Set of Real Numbers
- Solving Equations
- Solving Inequalities

Keywords

- Equations, integers, inequality, intervals, irrational numbers, natural numbers, rational numbers, real numbers, solution

Lesson 2. The Cartesian Plane

Code: C351G0SU01L02

Objectives

When completing this lesson, the student will:

- Apply the Pythagorean theorem to determine the distance between two points in the Cartesian plane.
- Find the middle point of a segment in the Cartesian plane.
- Find the equation of a circle in the Cartesian plane.
- Identify the center and radius of a circumference in the Cartesian plane.

Subjects

- Distance Formula
- The Middle Point
- The Circle Equation

Keywords

- Center, circle, circumference, distance, Cartesian plane, middle point

Lesson 3. Lines

Code: C351G0SU01L03

Objectives

When completing this lesson, the student will:

- Calculate and identify the constant rate of change (slope) of a line.
- Find the line equation given the slope and one or two points.
- Find the equation of the perpendicular line to a given line.

Subjects

- Constant Rate of Change
- Line Equation
- Parallel and Perpendicular Lines

Keywords

- Constant, parallel line, slope, perpendicular line, rate, line

Lesson 4. Functions

Code: C351G0SU01L04

Objectives

When completing this lesson, the student will:

- Establish if a relationship is a function.
- Determine the domain and image of a function.
- Evaluate a function.
- Sketch the graph for basic functions and its transformations.
- Perform operations of addition, subtraction, multiplication, and composition of functions.
- Find the inverse function.

Subjects

- Evaluation of Functions
- Domain and Image
- Graph of Functions
- Algebra of Functions
- Inverse Function

Keywords

- Image, composition, domain, evaluation, inverse function, transformations

Lesson 5. Trigonometric Functions

Code: C351G0SU01L05

Objectives

When completing this lesson, the student will:

- Find the trigonometric values of angles of 30, 45, and 60 degrees and their multiples.
- Sketch the graph of trigonometric functions of the sine, cosine, and tangent.
- Simplify trigonometric expressions by applying trigonometric identities.
- Solve triangles by applying inverse functions of trigonometric functions.

Subjects

- Angles and Unit Circle
- Trigonometric Functions
- Special Triangles $30°$, $60°$ and $45°$
- Trigonometric Functions with Angles Higher than $90°$
- Graph of Trigonometric Functions
- Trigonometric Identities
- Proof of Identities
- Addition and Subtraction Identities
- Identities of Double and Half Angle
- Inverse Functions
- Trigonometric Equations

Keywords

- Cosecant, cosine, cotangent, identities, inverse function, secant, sine, tangent, trigonometric values

Lesson 6. Exponential and Logarithmic Functions

Code: C351G0SU01L06

Objectives

When completing this lesson, the student will:

- Evaluate exponential functions.
- Sketch graphs of an exponential function.
- Solve exponential equations.
- Change an exponential expression to a logarithmic form and vice versa.
- Solve logarithmic equations.
- Sketch the graph of a logarithmic function.

Subjects

- The Exponential Function
- The Logarithmic Function
- Properties of Logarithms
- Equations with Logarithms
- Common Logarithms

- Natural Logarithms and Number *e*
- Exponential Equations
- Solving Problems Using Logarithms

Keywords

- Base, exponential equation, logarithmic equation, exponent, exponential function, logarithm

Unit 2. Limits and Continuity

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Concept of Function Limits

Code: C351G0SU02L01

Objectives

When completing this lesson, the student will:

- Understand the concept of function limits in a point.
- Understand the concept of side limits to determine when the limit of a function exists.
- Determine if the limit of a function is defined in a point.
- Develop a geometrical intuition of the side limits to calculate through graphs.
- Calculate the limit of a function in a point through its graph.
- Develop a numerical intuition of the side limits through a table of values.
- Calculate the limit of a function in a point through a table of values.

Subjects

- Limit Notation
- Limits on a Graph
- Limits in a Table of Values

Keywords

- Approximation, limits, side limit

Lesson 2. Limit Properties of the Function

Code: C351G0SU02L02

Objectives

When completing this lesson, the student will:

- Apply the properties of limits in order to calculate limits algebraically.
- Know limits of main basic functions.
- Apply known limits and properties of limits in order to find general limits of functions.
- Identify indefinite integrals and apply tools to simplify them.

Subjects

- Properties of the Constant and Identity
- Properties of the Power and Absolute Value
- Properties of the Constant Factor, Addition, and Subtraction
- Properties of Multiplication and Division

Keywords

- Indeterminate forms, limit properties, rationalize, simplify

Lesson 3. Trigonometric Limits

Code: C351G0SU02L03

Objectives

When completing this lesson, the student will:

- Identify indeterminate forms and strategies to simplify them.
- Evaluate limits related to indeterminate forms.
- Evaluate limits of trigonometric functions.
- Apply the squeeze theorem to evaluate the indeterminate forms.
- Identify special trigonometric limits.
- Apply special trigonometric limits to evaluate limits of trigonometric forms.

Subjects

- Indeterminate Forms and Strategies for Evaluating Limits
- Limits of Sine and Cosine
- Squeeze Theorem
- Limits of Trigonometric Forms

Keywords

- Evaluation, indeterminate forms, trigonometric limits, squeeze theorem

Lesson 4. Limits Related to Infinity

Code: C351G0SU02L04

Objectives

When completing this lesson, the student will:

- Find the limit of a function when x tends toward infinity.
- Apply the limit of a function to find vertical and horizontal asymptotes.

Subjects

- Vertical Asymptote
- Horizontal Asymptote

Keywords

- Horizontal asymptote, vertical asymptote, infinity

Lesson 5. Formal Definition of Limits

Code: C351G0SU02L05

Objectives

When completing this lesson, the student will:

- Define the limit of a function from epsilon and delta.
- Apply the theorem of differences between epsilon and delta to find the limit of a function.

Subjects

- Formal Definition of a Limit
- Proof of Limits
- Limits Applied by Infinity

Keywords

- delta, epsilon, infinity, limit

Lesson 6. Continuity of the Function

Code: C351G0SU02L06

Objectives

When completing this lesson, the student will:

- Define the continuity of a function in given values.
- Determine the continuity of a function by parts.
- Determine the continuity of an interval of a function.
- Identify the continuity of the addition, product, or cosine of functions.
- Determine the continuity of the inverse function.
- Find the zeros of a function by the bisection method.

Subjects

- Definition of Continuity
- Properties of a Continuous Function
- Continuity on an Interval: Intermediate Value Theorem

Keywords

- Continuity, inverse function, function by parts, interval, bisection method

Unit 3. Derivatives

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. The Slope of a Tangent Line

Code: C351G0SU03L01

Objectives

When completing this lesson, the student will:

- Apply the limit of the difference quotient when h tends toward zero to find the equation of the tangent line to the graph of a function.
- Find the equation of the tangent line at a point given by the graph of a function.

Subjects

- Average Rate of Change
- Difference Quotient
- Limit of the Difference Quotient

Keywords

- Difference quotient, limit of the difference quotient, average rate of change, secant line, tangent line

Lesson 2. The Derivative

Code: C351G0SU03L02

Objectives

When completing this lesson, the student will:

- Find the derivative of a function.
- Find the equation of the tangent line at a given point.
- Determine if a function is differentiable.

Subjects

- Definition and Notation of the Derivative
- Diferentiation and Continuity

Keywords

- Continuity, derivative, differentiation, tangent line

Lesson 3. Basic Rules of Differentiation

Code: C341G0SU03L03

Objectives

When completing this lesson, the student will:

- Derive a function applying the power rule.
- Derive the constant function.
- Apply the addition and difference rule to derive a function.

- Prove the product rule for derivatives.
- Find the derivative of a product.
- Prove the quotient rule for derivatives.
- Find the quotient derivative.

Subjects

- The Constant Rule
- Identity Rule
- Power Rule
- Addition and Subtraction Rule
- Product Rule
- Quotient Rule

Keywords

- Constant, derivative, identity, power rule, addition and subtraction rule, quotient rule, product rule

Lesson 4. Derivative of Transcendental Functions

Code: C351G0SU03L04

Objectives

When completing this lesson, the student will:

- Prove the derivative of sine and cosine.
- Derive trigonometric functions of sine and cosine.
- Prove the derivative of other trigonometric functions.
- Find the derivative of trigonometric functions of the tangent, cotangent, secant, and cosecant.
- Prove the derivative of an exponential function.
- Find the derivative of a natural exponential function.
- Apply the chain rule to find the derivative of exponential functions.
- Find the derivative of logarithmic functions.
- Apply guidelines to find logarithmic differentiation.

Subjects

- Derivative of Sine and Cosine
- Derivative of Tangent, Cotangent, Secant, and Cosecant.
- Derivative of Exponential and Logarithmic Functions.

Keywords

- Derivative, exponential function, logarithmic function, trigonometric function

Lesson 5. Tangent and Normal Line

Code: C351G0SU03L05

Objectives

When completing this lesson, the student will:

- Find the tangent line or curve alingment at a determined point.
- Use differentiation to approximate values about the rate of change of a tangent line at a given point of a function.

Subjects

- The Equation of the Tangent and Normal Line
- Relative Maxima and Minima

Keywords

- Alignment, curve, slope, rate of change, tangent

Unit 4. Differentiation of Composite, Implicit, and Inverse Functions

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Composite Functions

Code: C351G0SU04L01

Objectives

When completing this lesson, the student will:

- Find the composition of functions.
- Determinate the domain of composite functions.
- Analyze graphs of composite functions.

Subjects

- Composition of Functions
- Domain and Image
- Analysis of the Graphs for Composite Functions

Keywords

- Composition, domain, composite function

Lesson 2. The Chain Rule

Code: C351G0SU04L02

Objectives

When completing this lesson, the student will:

- Differentiate the power of a function.
- Prove the chain rule.
- Apply the chain rule to differentiate a function.

Subjects

- Chain Rule for Polynomial Functions
- Chain Rule for Trigonometric Functions
- Chain Rule for Exponential and Logarithmic Functions
- Higher Order Derivatives

Keywords

- Derivative, higher order, power, chain rule

Lesson 3. Implicit Differentiation

Code: C351G0SU04L03

Objectives

When completing this lesson, the student will:

- Use implicit differentiation to find the derivative of a function.
- Find the derivative of a function at specific points.

- Apply the implicit differentiation to resolve tangency and application problems.

Subjects

- Implicit Functions
- Differentiation of Implicit Functions

Keywords

- Composite, differentiation, implicit differentiation, chain rule, tangency

Lesson 4. Differentiation of Inverse Functions

Code: C351G0SU04L04

Objectives

When completing this lesson, the student will:

- Prove when a function is continuous or differentiable.
- Determine the continuity of a function.
- Find the derivative of inverse functions.
- Find the derivative of inverse trigonometric functions.

Subjects

- Derivatives of Inverse Functions
- Derivatives of Inverse Trigonometric Functions

Keywords

- Continuous function, differentiable function, inverse function

Unit 5. Application of the First and Second Derivative

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Intervals of Increase and Decrease

Code: C351G0SU05L01

Objectives

When completing this lesson, the student will:

- Determine the absolute extrema of a function.
- Find the relative extrema of a function.
- Identify the critical points (maximum and minimum) of a function.
- Find the critical points of a function at a defined interval.
- Prove Rolle's theorem relating it with the critical points of a function.
- Apply Rolle's theorem to find the maximum and minimum points of a function.
- Use the derivative to determine intervals of increase and decrease of a function.
- Calculate limits of indeterminate form.
- Apply L'Hôpital's rule to calculate limits of indeterminate forms.
- Apply L'Hôpital's rule consecutively.
- Apply the first derivative to determine the intervals of increase and decrease on the graph of a function.
- Find the relative critical points on a graph of a function.

Subjects

- Critical Points
- Intervals of Increase and Decrease
- Relative Maximum and Minimum Values
- Extreme Value Theorem and Absolute Extrema

Keywords

- Absolute extrema, interval, L'Hôpital's rule, relative maximum, relative minimum, critical points, Rolle's theorem, mean value

Lesson 2. Concavity and Inflection Point

Code: C351G0SU05L02

Objectives

When completing this lesson, the student will:

- Determine the concavity on the graph of a function at a given interval.
- Use the second derivative to find the inflection point of a function.
- Apply the first and second derivatives to optimize solutions for word problems.
- Find the maximum and minimum values of a function.

Subjects

- Concavity
- Inflection Point

Keywords

- Concavity, inflection point, interval, relative maximum, relative minimum, optimize

Lesson 3. Graph Analysis

Code: C351G0SU05L03

Objectives

When completing this lesson, the student will:

- Generate the graph of a function given its algebraic expression.
- Determine the information needed to graph a function.
- Analyze the graph of a function with the purpose of extracting information about its derivatives.
- Analyze the graph of the derivatives with the purpose of extracting information about the function.

Subjects

- Sketching Graphs
- Graphical Analysis

Keywords

- Critical points, inflection points, intercepts, tendency

Lesson 4. Application of the Derivative

Code: C351G0SU05L04

Objectives

When completing this lesson, the student will:

- Determine the position of a particle in motion from a position function.
- Define the speed and acceleration of a particle as a rate of change.
- Interpret algebraic signs of the speed and acceleration function of a particle.
- Determine the rate of change in application problems.
- Identify the related variables in a rate of change.
- Apply the derivative and power rule to resolve problems related to the rate of change.

Subjects

- Interpreting the Rate of Change
- Derivative, Speed, and Acceleration
- Related Rates of Change
- Limits of Indeterminate Forms and L'Hôpital's Rule

Keywords

- Acceleration, optimization, position function, rate of change, range of change, speed

Unit 6. Antiderivatives and Integrals

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Antiderivatives

Code: C351G0SU06L01

Objectives

When completing this lesson, the student will:

- Find the antiderivative of a function.
- Use the notation of integrals for expressing the antiderivative of a function.
- Determine the indefinite inegral of a function.
- Resolve simple differentiable equations.

Subjects

- Definition and Notation
- Basic Integration Rules
- Integration of Trigonometric Functions

Keywords

- Antiderivative, differentiable equations, integral notation, definite integral, indefinite integral

Lesson 2. Method of Substitution and Change of Variables

Code: C351G0SU06L02

Objectives

When completing this lesson, the student will:

- Analyze composite functions with the purpose of determining the inside and outside function.
- Calculate antiderivatives of composite functions using the method of substitution or change of variables.

Subjects

- Composite Functions
- Substitution Method

Keywords

- Antiderivative, composite, transcendental functions, substitution

Lesson 3. Integrals of Exponential and Logarithmic Functions

Code: C351G0SU06L03

Objectives

When completing this lesson, the student will:

- Analyze composite functions that involve the derivative of a logarithmic or exponential function.
- Calculate antiderivatives that involve exponential and logarithmic functions.

Subjects

- Integrals of Exponential Functions
- Integrals of Logarithmic Functions

Keywords

- Exponential integral, reciprocal integral, logarithmic integral

Lesson 4. Trigonometric Substitution

Code: C351G0SU06L04

Objectives

When completing this lesson, the student will:

- Identify the different forms of Pythagorean trigonometric identities.
- Distinguish between the possible trigonometric substitutions based on the expression present in the integrand.
- Evaluate integrals that result in inverse trigonometric functions.

Subjects

- Trigonometric Substitution

Keywords

- Pythagorean identities, trigonometric substitution

Lesson 5. Additional Integration Methods

Code: C351G0SU06L05

Objectives

When completing this lesson, the student will:

- Identify when is necessary to complete the square for evaluating an integral.
- Evaluate integrals using the method of completing the square.
- Perform long division of polynomials effectively.
- Evaluate integrals using the long division method.

Subjects

- Completing the Square
- Integration by Long Division

Keywords

- Quotient, completing the square, long division, remainder

Unit 7. Differential Equations

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Differential Equations

Code: C351G0SU07L01

Objectives

When completing this lesson, the student will:

- Identify the solution for a differential equation.
- Interpret direction field associeted to a differential equation.
- Resolve separable differential equations.
- Resolve linear differential equations.
- Resolves initial value problems.

Subjects

- Differential Equations and their Solutions
- Direction Fields
- Resolving Basic Differential Equations
- Separation of Variables
- Initial Value Problems

Keywords

- Slope field, differential equation, degree of a differential equation, initial value

Lesson 2. Applications of Differential Equations

Code: C351G0SU07L02

Objectives

When completing this lesson, the student will:

- Define the speed and acceleration of a particle as a rate of change.
- Determine the position of a particle from speed or acceleration functions.
- Apply initial value problems to simple harmonic motion situations.
- Resolve problems of growth and decay with applications to economy and population growth.

Subjects

- Linear Motion Problems
- Simple Harmonic Motion
- Applications to Economy
- Growth, Decay, and Logistic Equation

Keywords

- Exponential growth, exponential decay, tensor equation, logistic equation harmonic motion

Unit 8. Fundamental Theorem of Calculus

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. Summation Notation

Code: C351G0SU08L01

Objectives

When completing this lesson, the student will:

- Apply the rules of summation to simplify expressions.

Subjects

- Summations and their Properties
- Special Sums

Keywords

- sigma, summation

Lesson 2. Definite Integrals

Code: C351G0SU08L02

Objectives

When completing this lesson, the student will:

- Find the area under the curve of a function enclsoed by the axis x defined in an interval $[a, b]$.
- Use and simplify expression with the sigma sign (Σ).
  Find the area under the curve of a function using the limit of a summation.
- Calculate the Riemann sum for a determined function.
- Define the definite integral for an interval $[a, b]$.
- Determine if a function has integrability in an interval $[a, b]$.
- Find the area under the curve of a function as a defined integral.

Subjects

- Area Under the Curve
- The Riemann Sum
- Properties of the Definite Integral

Keywords

- Area, definite integral, interval, Riemann sum, sigma, summation

Lesson 3. Fundamental Theorem of Calculus

Code: C351G0SU08L03

Objectives

When completing this lesson, the student will:

- Prove the Fundamental theorem of calculus.
- Apply the fundamental theorem of calculus to find and evaluate definite integrals.
- Use the fundamental theorem of calculus in the derivative form.
- Apply the chain rule as an integration strategy.

Subjects

- Definition
- Proof
- Evaluation of Definite Integrals
- Trapezoidal Rule
- The Simpson Rule

Keywords

- Definite integral, interval, Simpson rule, trapezoidal rule, fundamental theorem of calculus

Unit 9. Area and Volumes of Revolution

When completing this unit, students will have completed all the objectives found in the following lessons:

Lesson 1. The Area of Regions

Code: C351G0SU09L01

Objectives

When completing this lesson, the student will:

- Find the area under the curve of a function enclosed by the axis x.
- Find the area between two curves of intersect functions.

Subjects

- Area Under the Curve of a Function
- Area Between Curves

Keywords

- Enclosed function, area, intersect functions, region

Lesson 2. Volumes of Revolution

Code: C351G0SU09L02

Objectives

When completing this lesson, the student will:

- Build integrals to find the volume of a three-dimensional body.
- Find the volume of a solid with the rectangular slicing method.
- Find the volume of a solid with the circlular slicing method.
- Find the volume of a solid with the ring slicing method.
- Build integrals to find the volume of a three-dimsensional body.
- Find the volume of a solid with the rectangular shell method.

Subjects

- The Disk Method
- The Ring Method
- The Cylindrical Method

Keywords

- The ring method, the shell method, the cylindrical method, the disk method, integral, the slicing method, solids of revolution, three-dimensional figure, volume

Lesson 3. Arc Length and Surface Area

Code: C351G0SU09L03

Objectives

When completing this lesson, the student will:

- Build integrals to find the arc length between two points of a function.
- Find the arc length of a segment in the graph of a function.
- Build integrals to find the surface area of a solid of revolution.
- Find the area of a solid of revolution.

Subjects

- Arc Length of a Curve
- Surface Area of a Solid of Revolution

Keywords

- Arc, length, revolution, segment, solid, surface

Lesson 4. Work, Energy, and Power

Code: C351G0SU09L04

Objectives

When completing this lesson, the student will:

- Use the integral to find the position or speed of an object departing from the speed or acceleration functions respectively.
- Apply the integral in solving word problems.
- Build an integral to calculate and determine the work done, expressed in a mathematical model.
- Apply the integral to calculate work in application problems.
- Building an integral to calculate and determine the pressure and force of a fluid expressed in a mathematical model.
- Apply the integral to calculate the pressure and force of a fluid in application problems.
- Build an integral to calculate the center of mass and the centroid of a solid expressed in a mathematical model.
- Apply the integral to calculate the center of mass and centroid in application problems.

Subjects

- The Work of a Pump
- Compression and Expansion of Gases
- Pressure and Force of Liquids
- Energy
- Gravity
- Consumers and Surplus

Keywords

- Pump, center of mass, circulation, compression, energy, power consumption, expansion, fluid, gas, surplus