DREYFOUS & ASSOCIATES

Course Description

Precalculus

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Course Description

The main objectives of the Precalculus course are to help students develop high level mathematical skills and to create awareness about the importance of studying mathematics in order to respond to problems and situations that emerge in day to day life. Through the developed content and the strategies and techniques used, we intend to bring about a deep understanding of the concepts, as well as the technical skills necessary for subsequent courses and their applications. The way the topics are introduced and presented, as well as the way the mathematical skills are developed through examples and applications, allow students to visualize, understand, and value their usefulness in daily life. The areas and topics discussed in the course include: functions, system of equations, matrixes, parametric equations, analytic geometry, vectors, polar coordinates, complex numbers, conic sections, sequences and series, and trigonometry, among others.

The content takes into account the *Estándares de Contenido y Expectativas por Grado* (Content Standards and Grade-Level Expectations, Puerto Rico Core Standards) of the Department of Education of Puerto Rico (2014) and the United States *Common Core State Standards*. The outline of objectives per lesson takes into account all of the necessary skills and concepts for students to establish connections between the different standards (numbering and operations, algebra, functions, geometry, measurement, and data analysis), into which mathematics are currently categorized. The learning approach is based on understanding concepts, developing skills, and solving mathematical problems, along with the development of critical thinking as a means for holistic development of students.

The course includes content related to science, technology, and engineering, among other disciplines, which serve a dual purpose: help students see the direct application of what they learn, and visualize the importance of mathematics as a universal discipline that serves society and its institutions. On the other hand, the inclusion of real life problems and situations in each of the topics will awaken the students' interest toward the discipline.

Course Structure

The Precalculus course consists of nine units that are carefully subdivided into several lessons. The amount of lessons per unit depends on the reach and the depth with which we discuss and develop different topics. Each unit begins with a brief video that exposes, in a practical manner, the importance of the topics to be discussed in everyday life. Each lesson has an interactive presentation which is divided into sections where we expose and explain the topic under study. In each presentation, there are definitions, concrete examples, explanations, simulations, representations with manipulatives, practice exercises, and application of concepts and skills used in daily life.

On the other hand, lessons also include practice exercises, quizzes, extra practice laboratory, homework, self-evaluation and a descriptive log with detailed information for the teacher, as well as a variety of Internet links and other resources. Some lessons include laboratories that present and reinforce algebraic and geometric concepts, through the use of manipulatives and technological tools like the graphing calculator and interactive activities. The activities are varied and flexible, with the purpose of satisfying the particular needs and interests of each student. The practice and self-evaluation activities aim to make students aware of their strengths and weaknesses in gaining command of the content, with the purpose of having the students gradually take control of their learning process. The teacher, as an essential part of the process, will have the responsibility of stimulating, counseling, guiding, and periodically evaluating the level of learning each student achieves.

The units are made up of the following parts:

Lessons

Each unit is made up of different lessons, divided by topics, macro-concepts, and skills. Simultaneously, each lesson is made up of five basic elements: presentation or lesson content, documents in digital format (PDF), Internet links, self-evaluation, and descriptive log.

- **Descriptive Log.** It's a detailed plan of the lesson. It includes the specific objectives, the content standards and grade-level expectations, the teaching strategies and resources, keywords, Internet links, and references. Only the teacher will have access to the descriptive log for each lesson.
- **Presentation (*Lesson Content*).** Each presentation contains a detailed explanation of the lesson's concepts and skills, as per the established objectives. Additionally, they are made up of the following elements that systematically contribute to the development of learning in students:
  - **Examples.** In each section, as skills are developed, we include examples that  explain, step by step, the solution of an exercise or problem, so that students can review concepts and skills.
  - **Practice.** It includes a series of exercises that have been carefully chosen to have students practice the skills and concepts under discussion. Its purpose is to verify  the level of learning students have reached before carrying on with other topics and skills. It does not include processes or explanations, only solutions.
  - **Solution.** It's used to keep the solution to an exercise or problem hidden while  students try to answer it. The solution is displayed when pressing the button.
  - **Process.** Label under which the steps or algorithm for solving an exercise or  problem appear.

- **Proof.** Label under which the steps or algorithm for solving an exercise or problem appear.

- **Calculator.** Includes an explanation of the processes in the use and management of a calculator, in order to solve the exercises in a section. It also connects the students to a virtual graphing calculator.

- **Animation.** Provides access to explanations, processes, or graphics that visually demonstrate the concepts and skills discussed in the section. They allow us to tend to students' development and conceptual understanding.

- **Definition.** Includes formal definitions of mathematical concepts and processes that have been mentioned or discussed.

- **Biography.** Includes a short biography of the mathematician or scientist we credit for the development of a definition, formula, process, or demonstration used in the lesson.

- **Note.** Under this icon, we discuss common mistakes or reinforce details that must not be forgotten.

- **Did You Know…?** It's a section that shows an explanation or situation that connects aspects of daily life with the skills and concepts discussed. In some cases, this section shows the link between the development of logical thought in human beings with certain skills and mathematical processes.

- **Tabs.** Tabs are located on the right side of the presentation, they can be red or blue. They unfold toward the left and provide flow charts, biographies, notes, pictures, explanations, suggestions, reminders, “Did You Know...,” or necessary previous knowledge.

- **Incorrect.** Indicates when the student has chosen an incorrect answer in the practice exercises.

- **Correct.** Indicates that the correct answer has been chosen in an exercise or practice problem.

- **Picture.** Connects a particular explanation to a picture that is probably accessed via Internet.

- **Video.** Gives you access to a short video that links the mathematical concept to daily life.

- **Internet.** It’s a direct link to a page that is closely related to the topic.

Each of the sections in the presentation are linked to a particular icon that identifies it with its explanation. In the initial presentations of the course, we include an icon with a word that describes each section so that students can become familiar with what each of the icons represent. In subsequent presentations, it only includes the icon that provides access to the section. Pressing the icon will take the presentation to the section it represents.

- **PDF Documents.** These documents include a copy of the practice exercises in the lesson, an additional practice section, activities to work on with a calculator, or homework. These documents can be printed so that students can work on them with pencil. The homework

documents are exercises and problems that students can work on at home and allow them, through practice, to refine their newly acquired skills and concepts. Homework is optional.

- **Internet Links.** These links are a direct connection to the Internet and can be accessed from the presentation. They include additional explanations, examples, applications, or demonstrations that allow students to conceptually develop the skills and topics discussed.
- **Auto-evaluation.** It consists of practice quizzes that the student answers to monitor his or her own progress before taking the teacher's formal evaluation test for the unit.

Unit Documents

- **Lesson 0.** It is not a lesson that provides content, but rather a file with elements inside the Unit: PDF documents, unit evaluations, and introductory video.
    - **PDF Documents.** Documents with laboratory activities for application and conceptual development, laboratory activities for the development of skills, activities using a calculator, extra exercises, and assessment activities.
    - **Evaluations.** This includes the evaluations to be used in the unit: the unit pretest and post-test. Lesson 00 of Unit I includes a diagnostic test for the entire course.
    - **Real Zone.** It consists of a short video that presents situations or elements in our immediate environment, in which we concretely visualize the use and application of concepts and skills to be studied in the unit. In this video, we can introduce the topic to be studied, presenting the usefulness in daily life of what the student will learn.

List of Units

Below, you will find an itemized list of the course's content, including its respective units, lessons, general objectives by unit, the subject or title of each unit and lesson, as well as the lesson specific objectives and concepts.

Unit 1. Fundamentals of Algebra

This is an introductory unit which generally, yet widely, reviews all the skills of an algebra course that are necessary to properly enter the field of precalculus. These skills include: number sets, operations with real and complex numbers, order of operations, algebraic expressions, operations with polynomials, the resolution of equations and inequalities with or without absolute value, irrational equations, distance and midpoint, linear equations, slope, and relations between lines on a plane.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

General objectives

- Define and classify numbers into different number sets: natural, whole, integers, rational, irrational, real, and complex.
- Solve exercises regarding addition, subtraction, multiplication, and division of real and complex numbers.
- Solve exercises by applying the order of operations.
- Simplify algebraic expressions by using the laws of exponents.
- Simplify rational expressions.
- Solve exercises regarding the addition, subtraction and factorization of polynomials.
- Solve absolute value and irrational equations, inequalities, and expressions.
- Determine distance and midpoint.
- Apply rate of change to determine the slope of a line.
- Determine the equation of a line and solve problems with applications in parallel and perpendicular lines.

Lesson 0. Fundamentals of Algebra

Code: C325G0SU01L00

Unit Documents

Lesson 1. Real Numbers Set

Code: C325G0SU01L01

Objectives

- Define different number sets and classify numbers as part of these sets.

- Express sets in different ways: list of elements, set notation, graphically, or interval notation.
- Evaluate operations of union and intersection of sets.
- Define the concept of absolute value as a distance.
- Determine the distance between two points.
- Solve problems that have an absolute value.

Keywords

- absolute value
- distance
- empty set
- intersection
- interval
- irrational number
- periodic decimals
- precalculus
- sets
- union

Lesson 2. Properties of Exponents

Code: C325G0SU01L02

Objectives

- Define the exponential forms and express them from their developed form to their exponential form and vice-versa.
- Apply the laws and properties of exponents to simplify algebraic expressions.

Keywords

- exponent
- exponential notation
- laws
- multiplicative inverse
- power
- properties
- reciprocal

Lesson 3. Order of Operations and Algebraic Expressions

Code: C325G0SU01L03

Objectives

- Simplify expressions by applying the order of operations.
- Evaluate operations with polynomials.
- Apply the rules of notable products to factor polynomials.

Keywords

- algebraic expression
- binomial

- coefficient
- degree of a polynomial
- degree of a term
- evaluate
- like terms
- notable products
- polynomial
- terms
- trinomial

Lesson 4. Factorization

Code: C325G0SU01L04

Objectives

- Break down a number or an expression as a product of factors.
- Factor polynomials via common factor, grouping, or applying the rules of notable products.

Keywords

- difference of cubes
- difference of squares
- factor
- factorization
- notable products
- polynomials
- trinomials

Lesson 5. Rational Expressions

Code: C325G0SU01L05

Objectives

- Define rational algebraic expressions.
- Establish for which value or values a rational expression is not defined.
- Evaluate operations with rational algebraic expressions.
- Simplify rational expressions with terms that are made up of complex fractions.

Keywords

- complex fractions
- factor
- multiplicative inverse
- rational expression
- simplify

Lesson 6. Expressions with Radicals

Code: C325G0SU01L06

Objectives

- Define a radical as an exponential expression with a fractional exponent and convert it from its radical form to its exponential form and vice-versa.
- Apply the properties and laws of whole exponents we studied earlier to fractional exponents.
- Simplify radical expressions.
- Rationalize fractions whose terms are expressions with radicals, using the conjugate of the denominator.
- Define imaginary numbers as even roots of negative numbers, where $i = \sqrt{-1}$, and complex numbers as numerals with a real part and an imaginary one.
- Evaluate operations with complex numbers.

Keywords

- complex number
- conjugate
- imaginary number
- index
- irrational number
- radical
- radicand
- rationalize

Lesson 7. Solving Linear Equations and Absolute Value

Code: C325G0SU01L07

Objectives

- Solve linear and simplifiable equations as linear equations.
- Solve equations that contain absolute value and represent the solution set in interval notation and graphically.
- Translate verbal expressions into algebraic expressions and vice-versa.
- Use a formula for different variables.

Keywords

- absolute value
- distance
- formula
- null or empty set
- set notation
- solution set

Lesson 8. Solving Linear Inequalities and Absolute Value

Code: C325G0SU01L08

Objectives

- Define and solve inequalities with a single or double sign of inequality.
- Apply the principle of change in direction of the inequality sign when multiplying or dividing by a negative value on both sides of the inequality.
- Determine the viability of the solutions in inequality exercises that involve absolute value.

Keywords

- change in direction of the inequality
- closed interval
- inequality
- interval notation
- open interval
- signs of inequality

Lesson 9. Solving Quadratic Equations by Factoring

Code: C325G0SU01L09

Objectives

- Factor different types of trinomials.
- Solve quadratic equations through factorization.
- Solve quadratic equations in a concrete manner by using manipulatives (algebraic tiles).

Keywords

- factorization
- null product
- quadratic equations
- quadratic trinomials

Lesson 10. Solving Quadratic Equations by Completing the Square

Code: C325G0SU01L10

Objectives

- Solve simple quadratic equations by applying the square root.
- Apply the rules of notable products to develop the method of completing the square.
- Solve quadratic equations by completing the square.

Keywords

- completing the square
- manipulatives
- quadratic binomial
- square root
- trinomial

Lesson 11. Solving Quadratic Equations by Using the Quadratic Formula

Code: C325G0SU01L11

Objectives

- Transform quadratic equations to their general form: $ax^2 + bx + c = 0$
- Use the quadratic formula: $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to find solutions of quadratic equations.
- Apply the discriminant to determine the number and type of solutions for a quadratic equation.

Keywords

- coefficient
- complete the square
- discriminant
- quadratic formula

Lesson 12. Solving Quadratic Inequalities

Code: C325G0SU01L12

Objectives

- Apply the factorization of quadratic equations in solving inequalities.
- State the values of intervals in an inequality for which the inequality is true.
- State the solution set of quadratic inequalities.

Keywords

- inequalities
- intervals
- numbers of a proof
- sign diagrams

Lesson 13. Solving Equations with Radicals

Code: C325G0SU01L13

Objectives

- Review the properties of operations with roots.
- Solve equations with different types of radicals.
- State the rationality of solutions of irrational equations.

Keywords

- fractional exponents
- irrational equations
- laws of exponents
- radicals

Lesson 14. The Distance and Midpoint Formulas

Code: C325G0SU01L14

Objectives

- Determine the midpoint between two values on a number line.
- Determine the midpoint between points on a Cartesian plane.
- Determine the formula for distance on a plane through the application of the Pythagorean Theorem.
- Find the distance between two points on a plane.

Keywords

- coordinates
- distance
- distance on a plane
- hypotenuse
- midpoint
- Pythagorean Theorem

Lesson 15. Equation for a Circle's Circumference

Code: C325G0SU01L15

Objectives

- Develop the equation of a circle given the graph by applying the Pythagorean Theorem.
- Determine the radius of a circle given the equation and center.
- Determine the coordinates of the center of a circle given the equation.
- Determine the equation of a circle in different circumstances.

Keywords

- center
- circle
- circumference
- conic section
- diameter
- radius

Lesson 16. Rate of Change and the Slope

Code: C325G0SU01L16

Objectives

- Define the concept of rate of linear and nonlinear change.
- Establish negative and positive rates of change and as increasing and decreasing, respectively.
- Establish the rate of linear change as the slope of a line.
- Apply and explain the concept of the slope of a line to everyday situations.

Keywords

- constant
- decreasing

- difference
- direct relation
- increasing
- inverse relation
- linear relation
- rate of change
- sequence
- slope

Lesson 17. Linear Equations

Code: C325G0SU01L17

Objectives

- Determine the slope of a line given two points, using the formula of a slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Explain the implications of $m > 0$ (increasing line), $m < 0$ (decreasing line), $m = 0$ (line parallel to x), and $m =$ indeterminate (line parallel to y).
- Determine and explain what the y-intercept on a line represents.
- Find the equation of a line under different circumstances.
- Determine the point-slope, point-intercept, and general equations of a line, according to each case.

Keywords

- constant change
- equation of a line
- intercept
- point-slope equation
- rate of change
- slope
- slope-intercept equation

Lesson 18. The General Form of a Linear Equation

Code: C325G0SU01L18

Objectives

- Represent linear relations as table of values, graphs, and equations.
- Establish equivalences between different forms of linear equations: $y = mx + b$, $y_2 - y_1 = m(x_2 - x_1)$, and $Ax + By + C = 0$.
- Define and determine the intercept of the axes in a linear equation.
- Transform linear equations into one equivalent form or another.

Keywords

- general equation of a line

- intercept
- linear relation
- representation
- table of values

Lesson 19. Parallel and Perpendicular Lines

Code: C325G0SU01L19

Objectives

- Define and determine when two lines are parallel and when they are perpendicular.
- Determine the equation of a line given its relation of parallelism or perpendicularity with another line.
- Solve everyday problems by applying the relation between parallelism and perpendicularity of lines.

Keywords

- inverse multiplicative
- oblique lines
- parallel lines
- perpendicular lines
- reciprocal

Unit 2. Functions and Graphs

In the world around us, there are several relationships between two variables that can be expressed mathematically. In science, engineering, business, technology, and mathematics, among others, many of these relationships between variables are considered functions. The concept of functions is one of the most important notions in mathematics. In this unit, we will present and discuss the types of relationships between variables and which of them meet the criteria to be considered functions. In this unit we will also study function notations; graphs of functions and their transformations (linear, quadratic, cubic, absolute value, square root); characteristics of functions (increasing, decreasing, domain, field of values, symmetry, even and odd functions); algebraic functions, and inverse functions.

General Objectives

- Define the different types of relations between sets and identify real-world examples.
- Establish which of the relations between sets are considered functions.
- Evaluate, transform, and carry out operations with functions.
- Determine inverse functions.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Functions and Graphs

Code: C325G0SU02L00

Unit Documents

Lesson 1. Relations and Functions

Code: C325G0SU02L01

Objectives

- Identify and define relations and functions.
- Locate ordered pairs on the Cartesian plane.
- Determine the domain and the table of values of a function.

Keywords

- Cartesian plane
- coordinate
- domain
- function
- relation
- table of values

Lesson 2. Evaluation of Functions

Code: C325G0SU02L02

Objectives

- Evaluate expressions and values in functions.
- Define the difference quotient as the rate of change of a tangent line to a graph and find the difference quotient of a function.
- Determine the increasing, decreasing, and constant intervals in linear relations.
- Define the maximum and minimum points in a graph as rates of constant change.

Keywords

- constant
- decreasing
- difference quotient
- increasing
- interval
- rate of change
- tangent line
- variation

Lesson 3. Graphs of Functions

Code: C325G0SU02L03

Objectives

- Determine whether or not a graph is a function by applying the vertical line test.
- Solve everyday problems represented as a linear function and linear functions by parts.
- Determine increasing, decreasing, and constant intervals in linear relations.

Keywords

- constant
- decreasing
- increasing
- interval
- linear function by parts
- restricted domain
- variation

Lesson 4. Linear Transformations and Absolute Value

Code: C325G0SU02L04

Objectives

- Define and classify the different transformations in the graph of a function as translations, reflections, dilations.
- Identify and explain the transformations the graph of a linear function suffers from the identity function.
- State and explain the transformations suffered by a graph of a function of an absolute value.

Keywords

- absolute value
- dilations
- horizontal
- identity function
- reflection
- transformations
- translations
- vertical

Lesson 5. Transformations of Nonlinear Functions

Code: C325G0SU02L05

Objectives

- Apply the concepts of transformations (translations, reflections, dilations) to a linear function.
- Draw graphs of nonlinear functions that have undergone one or more transformations.
- Identify and explain the type of transformation a function has undergone from its graph.
- Solve everyday problems that involve operations with functions.

Keywords

- compression
- cubic function
- define the domain
- domain
- square root function
- intersection
- quadratic function

Lesson 6. Operations with Functions

Code: C325G0SU02L06

Objectives

- State the domains of functions in their different representations (table of values, ordered pairs, graph, and equation) and in the form of sets.

- Carry out additions and subtractions of functions and establish the domain of the remaining function.
- Carry out multiplications between functions and establish the domain of the product.

Keywords

- domain
- equation
- intersection
- ordered pairs
- restricted domain
- resulting function
- sets
- table of values

Lesson 7. Composition of Functions

Code: C325G0SU02L07

Objectives

- Define the composition of functions as an operation between functions.
- State and establish that the composition of functions is not a commutative operation.
- Carry out operations of compositions between functions.
- State the domain of the function that results from a function composition.

Keywords

- commutative
- composition of functions
- domain
- operation

Lesson 8. Inverse Functions

Code: C325G0SU02L08

Objectives

- Define *inverse function* and establish the symbols used to refer to them.
- State the inverse of a given function.
- State the domain of an inverse function.
- State when two functions are the inverse of the other.

Keywords

- domain
- identity function
- inverse function
- range of values

Unit 3. Polynomial and Rational Functions

In the previous unit, we worked with a wide array of functions classified as polynomial but from a monomial or binomial viewpoint and with one of the constant terms. In this unit we will work with polynomial functions that have two terms or more, that are different to the ones we discussed previously. Now we will start investigating the graphs of other quadratic functions and with other powers, which are the elements that make up polynomial functions. Then, we will work with synthetic division as a method to shorten the division of polynomials, which makes evaluating polynomial functions easier. We also include methods for determining real and complex zeros in polynomial functions. Finally, we will focus the attention on analysis and discussion of the graphs of real rational functions.

General Objectives

- Evaluate the values in a quadratic function, state its vertex, y-intercept, zeros in the function, concavity, and trace its graph.
- Define the polynomial function and state its critical points, y-intercepts, zeros in the function, relative maximums and minimums, and trace the graph.
- Establish the graphic and algebraic conditions to know if a function is even or odd, determine it through algebraic manipulation by using the graph of a function.
- State the quotient and remainder of a polynomial division through long division or synthetic division.
- Evaluate a polynomial function using the remainder theorem.
- Use the factor theorem to determine if an expression in $x - c$ form, is a factor of a polynomial function.
- State how many of the possible rational zeros in a polynomial function are x-intercepts.
- Apply Descartes' rule of signs to state the possible real and imaginary zeros in a polynomial function.
- Establish the domain of a rational function, determine its x and y-intercepts, determine its asymptotes, and trace the graph.
- Determine when a rational function has a diagonal asymptote and trace the graph.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Polynomial and Rational Functions

Code: C325G0SU03L00

Unit Documents

Lesson 1. General Quadratic Functions

Code: C325G0SU03L01

Objectives

- Determine the y-intercepts, zeros in the function, vertex, and concavity in quadratic functions.

- Determine the nature of the zeros in the quadratic function using the discriminant.
- Trace the graph of a quadratic function using the intercepts in the axes and vertex.

Keywords

- concavity
- discriminant
- intercepts
- vertex
- zeros in the function

Lesson 2. Standard Quadratic Functions

Code: C325G0SU03L02

Objectives

- Determine the vertex, intercept, concavity, and zeros in a quadratic function given its standard equation.
- Complete the square to transform a quadratic equation from general form to standard form.

Keywords

- concavity
- intercept
- standard form of the quadratic equation
- zeros

Lesson 3. Polynomial Functions

Code: C325G0SU03L03

Objectives

- Define and identify polynomial functions.
- Determine if a function is even or odd through algebraic manipulation, or by establishing the type of symmetry of the graph.
- Determine x, y, and critical point (maximum and minimum relatives, points of inflection), on the Cartesian plane.
- Determine variations in the graph, given a multiplicity of zeros.

Keywords

- critical points
- even functions
- maximum point
- minimum point
- odd function
- point of inflection
- variation
- zeros

Lesson 4. Division Algorithm and Synthetic Division

Code: C325G0SU03L04

Objectives

- Determine the quotient and remainder of a division of polynomials through the division algorithm or through synthetic division.
- Transform a rational expression to the form: $f(x) = Q(x)(x - c) + R$.

Keywords

- algorithms
- quotient
- remainder
- synthetic division

Lesson 5. Remainder Theorem and Factor Theorem

Code: C325G0SU03L05

Objectives

- Evaluate polynomial functions using the remainder theorem.
- Use the factor theorem to determine if an expression in the form $x - c$, is a factor of a polynomial function.

Keywords

- factor
- polynomial
- quotient
- remainder
- theorem

Lesson 6. Rational Zeros

Code: C325G0SU03L06

Objectives

- Determine the set of possible rational zeros in a polynomial function.
- Use the remainder theorem and synthetic division to determine what elements of a set of rational zeros are in the x-intercept of the graph of a polynomial function.
- Determine the number of zeros in a polynomial function and which of them are intercepts in the graph's abscissa.

Keywords

- abscissa
- intercepts
- rational zeros
- remainder
- synthetic division

Lesson 7. Irrational Zeros

Code: C325G0SU03L07

Objectives

- Use Descartes' rule of signs to determine the possible real and imaginary zeros.
- Determine between which two whole numbers the real zero of a polynomial function is located.
- Approximate the irrational zeros in a polynomial function to the nearest hundredth.

Keywords

- abscissa
- approximate
- bisection method
- factor
- irrational number
- irrational zero
- midpoint
- roots of a function
- rule of signs

Lesson 8. Complex Zeros

Code: C325G0SU03L08

Objectives

- Use the linear interpolation method to approximate the irrational zeros of a polynomial function.
- Determine the complex zeros of a polynomial function.
- Determine the polynomial function of complex zeros.

Keywords

- approximate
- bisection
- complex number
- imaginary number
- irrational number
- irrational zero
- linear interpolation

Lesson 9. Rational Functions

Code: C325G0SU03L09

Objectives

- Determine the domain of a rational function.
- Identify the y-intercept and the x-intercept in a rational function.
- Find the vertical and horizontal asymptotes of a rational function.
- Draw the graph of a rational function.

Keywords

- asymptote
- horizontal asymptote
- hyperbola
- rational function
- restricted domain
- sign chart
- undefined domain
- vertical asymptote

Lesson 10. Diagonal Asymptotes

Code: C325G0SU03L10

Objectives

- Determine the horizontal asymptote of a function using the limit when x tends toward the infinite.
- Identify when a rational function has a diagonal asymptote.
- Determine the equation of the diagonal asymptote.
- Draw the graph of the special cases of the rational function.

Keywords

- asymptote
- diagonal asymptote
- domain
- horizontal asymptote
- hyperbola
- infinite
- intercepts
- limit
- rational function
- vertical asymptote

Unit 4. Exponential and Logarithmic Functions

In this unit we will present two types of functions which closely relate to one another: exponential functions and logarithmic functions. These functions are classified as transcendental functions because they establish relations between variables that cannot be expressed as a polynomial equation. Both functions model everyday situations from different areas of biology, chemistry, economics, business, and engineering. The exponential function is used to model processes such as the population growth of viruses and bacteria, radioactive decay, and compound interest. An exponential function has an independent variable (x) where the exponent is located. Logarithmic functions are the inverse of exponential functions.

Logarithms have been used for many years because through them, we can shorten computational processes that make the work of astronomers and engineers easier. In the last decades, many logarithmic computations were carried out using logarithmic charts, using processes of extrapolation of values. Nowadays, with technological advances, those processes are not necessary since calculations of computations have become faster and more precise with the use of calculators.

General Objectives

- Define the exponential function with its domain and field of values.
- Evaluate the exponential function for different bases and establish the characteristics for the two types of graphs: $f(x) = b^x, b > 1,$ and $f(x) = b^x,\ 0 < b < 1.$
- Trace the graph of the exponential function for different bases and with different transformations.
- Define the logarithmic function as the inverse of the exponential function.
- Establish equivalent expressions between exponential and logarithmic forms: $b^y = x \ \equiv y = \log_b x.$
- Establish logarithmic identities and solve exponential equations by applying these identities.
- Evaluate the logarithmic function and trace its graph.
- Trace graphs of logarithmic functions with different transformation.
- Solve problems of the application of exponential and logarithmic functions in science, engineering, and finance.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Exponential and Logarithmic Functions

Code: C325G0SU04L00

Unit Documents

Lesson 1. Exponential Functions

Code: C325G0SU04L01

Objectives

- Define an exponential function as one in the form $b^x, b > 0$, and $b \neq 1$; establish their domain, $x \in \mathbb{R}, \{x| \in \mathbb{R}\}, (-\infty, \infty)$; their field of values, $y \in \mathbb{R}, y > 0$, and $y = 0$ as the horizontal asymptote.
- Evaluate the exponential function for the different values of x (exponent), and with different bases (b) and trace the graph.
- Define the number e and trace the graph of the exponential function in base e.
- Establish general characteristics for an exponential function $f(x) = b^x,\ b > 1$ and for exponential function $f(x) = b^x,\ 0 < b < 1$.
- Trace graphs for an exponential function with different transformations: narrowness and reflection.

Keywords

- asymptote
- base
- base e
- exponent
- exponential function
- number e

Lesson 2. Logarithmic Functions

Code: C325G0SU04L02

Objectives

- Establish the exponential function as a one-to-one function, and the logarithmic function as the inverse of the exponential function.
- Define the logarithmic function as $f(x) = \log_b x$, for $b > 0$, and $b \neq 1$.
- Convert exponential expressions into logarithmic functions and vice versa.
- Solve exponential equations.
- Find logarithm of numbers with different bases using the calculator.
- Trace the graph of logarithmic functions with different bases, including the logarithmic function with base e (or natural logarithm) $f(x) = \ln x$.
- Trace graphs of a logarithmic function with different transformations (translations and reflection), and establish the domain, field of values, and asymptote.

Keywords

- asymptote
- base e
- logarithm
- logarithmic function
- natural logarithm
- number e

Lesson 3. Properties of Logarithms

Code: C325G0SU04L03

Objectives

- Use the properties of exponents to derive properties of logarithms.
- Define properties of the logarithms of the product, quotient, and potentiation.
- Apply the definition of logarithm to establish the following equivalences: the logarithm of 1 is 0 ($\log_b 1 = 0$); the logarithm of the base is 1 ($\log_b b = 1$); the logarithm of the exponential form of the base is the exponent ($\log_b(b^x) = x$), and the base elevated to the logarithm of the base of a number is the number ($b^{(\log_b x)} = x$).
- Apply the properties of logarithms in the solution of exercises, including the natural logarithm $(\ln)$.
- Solve logarithmic equations converting them to their exponential form, and apply the change of base formula to find logarithms.
- Trace the graph of the logarithmic function with different transformations.

Keywords

- base
- logarithm
- exponentiation
- product
- properties
- quotient

Lesson 4. Financial Applications Involving Exponential and Logarithmic Functions

Code: C325G0SU04L04

Objectives

- Develop a conceptual definition of compound interest, effective annual interest, and interest calculated at the end of the year.
- Establish the formula for determining compound interest rate, applying lessons from exponential forms.
- Solve problems about compound interest.
- Define present value and future value and determine the formula for calculating the present value of an investment based on expected future value.
- Establish the formula for determining an approximate monthly payment for a loan.

Keywords

- compound interest
- effective annual interest
- future value
- loan
- present value

Lesson 5. Scientific Applications Involving Exponential and Logarithmic Functions

Code: C325G0SU04L05

Objectives

- Analyze the application of logarithmic and exponential applications in different scientific situations such as the pH of substances ($\mathrm{pH} = -\log[\mathrm{H}^+]$); volume of a sound with its intensity according to which the Weber- Fechner law is established, sound levels (sonority) of a sound in relation to hearing and human tolerance: $L = 10 \log \frac{I}{I_0}$, the intensity of an earthquake according to the Richter scale: $M = \log \frac{I}{I_0}$.
- Analyze the application of the logarithmic and exponential function in situations of disintegration (decay or half-life) of a radioactive substance, applying the decay equation: $y = y_0 e^{-kt}$.
- Apply what you learned about logarithmic and exponential function to everyday life: healing time of a wound, Newton's Law of Cooling: $T = T_0 + Ce^{-kt}$, and the formula for learning curve: $y = c - e^{-kt}$.
- Solve exponential formula application problems to different scientific situations.

Keywords

- half-life
- intensity of sound
- learning curve
- magnitude of an earthquake
- Newton's Law of Cooling
- pH
- radioactive decay
- sound levels
- tolerance scale

Unit 5. Fundamentals of Trigonometry

Beyond the study of polynomial, exponential, and logarithmic functions, there are other types of important functions: *trigonometric functions*. The development of trigonometry has its roots with the mathematicians and astronomers of ancient Greece and Egypt, especially with Greek astronomer and mathematician Hipparchus of Nicaea (2nd century BC), who we credit for the invention of trigonometry. He defined trigonometric functions as the rations between the length of the chord and its radius.

Nowadays we recognize two points of view for the use of trigonometric functions in research. The first approach proposes the use of the unit circle to define trigonometric functions for real numbers, which allows for a mathematical model that is applicable to the study of cyclical or periodic phenomena in electronics, engineering, physical science, and biological science. The second approach establishes that the trigonometric functions establish a relationship between angles and sides in triangles, and are also important for explaining real life situations in land surveying, engineering, and navigation.

General Objectives

- Define trigonometric functions based on the unit circle using a point $P(x, y)$ in the circumference determined by t.
- Associate the coordinates of the points in a unit circle to values in the real number set determined by a distance t (arc of the circumference).
- Define the domain and field of values for trigonometric functions.
- Evaluate trigonometric functions.
- Establish trigonometric identities.
- Calculate all of the trigonometric functions based on a single one.
- Express a trigonometric function based on a single one.
- Trace graphs of trigonometric functions.
- Apply the trigonometric functions to real life problems that present periodicity.
- Establish and determine amplitude and the period in sine and cosine functions.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Fundamentals of Trigonometry

Code: C325G0SU05L00

Unit documents

Lesson 1. Fundamentals of Trigonometry

Code: C325G0SU05L01

Objectives

- Define a circle with radius 1 in the Cartesian coordinate system.
- Define the standard position of angles on a rectangular coordinate system.
- Determine the coordinates of points around a unit circle in radians terms.

- Establish the equivalences between angle degree fractions within a decimal system of minutes and seconds, and express decimal degrees in terms of degrees, minutes, and seconds, and vice versa.
- Establish the equivalence between radians and degrees, and change from radians to degrees and degrees to radians.

Keywords

- angle
- angular measurement
- arc
- degrees
- measurement of angles in the decimal system
- radians
- terminal side
- unit circle

Lesson 2. Trigonometric Functions

Code: C325G0SU05L02

Objectives

- Define trigonometric functions for real numbers as circular functions with: $t\epsilon\mathbb{R}, P(t) = (x, y)$: $\sin t = y;\ \cos\ t = x;\ \tan t = \frac{y}{x}, x \neq 0; \csc t = \frac{1}{y}, y \neq 0; \sec t = \frac{1}{x}, x \neq 0; \cot\ t = \frac{x}{y}, y \neq 0.$
- Determine the six trigonometric functions for a pair of coordinates (x, y) given a value for t.
- Define the trigonometric functions for angles: $\sin\theta = \frac{y}{r}; \cos\theta = \frac{x}{r}; \tan\theta = \frac{y}{x}, x \neq 0; \csc\theta = \frac{r}{y}, y \neq 0; \sec\theta = \frac{r}{x}, x \neq 0; \cot\theta = \frac{x}{y}, y \neq 0.$
- Evaluate the trigonometric functions in an angle, given certain coordinates.
- Determine the trigonometric functions in an angle, given an equation of the line in the terminal side of an angle.

Keywords

- angle functions
- circular functions
- cosecant
- cosine
- cotangent
- secant
- sine
- tangent
- terminal side
- trigonometric functions

Lesson 3. Properties of Trigonometric Functions

Code: C325G0SU05L03

Objectives

- Determine the trigonometric functions of angles.
- Establish the trigonometric functions for the special angles $30°$, $45°$, and $60°$.
- Define the reciprocal identities of trigonometric functions.
- Establish whether the value of a trigonometric function of an angle is positive or negative according to the terminal side of an angle.

Keywords

- coterminals angle
- exact value
- standard position angle

Lesson 4. Evaluation of Trigonometric Functions

Code: C325G0SU05L04

Objectives

- Establish the trigonometric functions of triangles.
- Determine the trigonometric functions in right triangles, given their sides or legs.
- Determine the trigonometric functions in right triangles, given their angles.
- Define special or reference right triangles ($45° - 45°$ and $30° - 60°$) and use them to evaluate the trigonometric functions for a given angle.
- Apply trigonometric relationships to other triangles.
- Solve engineering problems by applying trigonometric relationships of triangles.

Keywords

- acute angle
- adjacent side
- angle of depression
- angle of elevation
- hypotenuse
- opposing side
- right triangle
- sides
- standard position of an angle
- trigonometric relationships

Lesson 5. Introduction to the Trigonometric Identities

Code: C325G0SU05L05

Objectives

- Establish that an identity is represented by an equation that is true for all values of a variable, for which both sides of an equation are defined.
- Define and demonstrate the trigonometric identity of the quotient: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$.
- Define and demonstrate the trigonometric identity of the Pythagorean identity for an angle in standard position, with the terminal side intersecting a unit circle in point (x, y): $\cos^2\theta + \sin^2\theta = 1,\ 1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$.
- Define and demonstrate the trigonometric identities, even and odd, related to the trigonometric functions of angle θ and $-\theta$: $\sin(-\theta) = -\sin\theta, \cos(-\theta) = \cos\theta,\ \tan(-\theta) = -\tan\theta, \cot(-\theta) = -\cot\theta,\ \sec(-\theta) = \sec\theta$, and $\csc(-\theta) = -\csc\theta$.
- Solve exercises by applying the fundamental trigonometric identities using a calculator.
- Simplify trigonometric expressions using essential identities.
- Determine the exact values of trigonometric expressions of angles in degrees and in radians using the even and odd identities of the trigonometric functions, without the calculator.

Keywords

- essential identities
- even and odd identity
- exact value
- Pythagorean identity
- quotient identity
- standard position of the angle

Lesson 6. Graphs of the Sine and Cosine Functions

Code: C325G0SU05L06

Objectives

- Establish the periodic properties of sine and cosine functions: period, amplitude, and phase change.
- Draw graphs of sine and cosine functions and establish their period and amplitude.
- Define the graphs of the sine and cosine functions as a type of wave called simple harmonic motion.
- Establish the characteristics of simple harmonic motion and determine the amplitude, angular frequency, and phase angle in these functions.
- Define frequency of oscillation and establish its formula.
- Determine the frequency of oscillation given the conditions of a harmonic motion.

Keywords

- amplitude
- angle phase
- angular frequency
- frequency of oscillation
- harmonic motion
- period
- phase change
- reach

Lesson 7. Graphs of Other Trigonometric Functions

Code: C325G0SU05L07

Objectives

- Define the tangent, cotangent, secant, and cosecant graphs based on the sine and cosine functions.
- Establish the vertical asymptotes for each of the graphs.
- Explain the graphs of other trigonometric functions in different intervals of the domain and the relationship with their reciprocal function.
- Determine which of the tangent, cotangent, secant, or cosecant functions is even or odd from its graph.

Keywords

- asymptotes
- cosecant
- cotangent
- reciprocal function
- secant
- tangent

Lesson 8. Inverse Trigonometric Functions

Code: C325G0SU05L08

Objectives

- Establish that none of the trigonometric functions has an inverse due to its condition of periodicity and its graph, unless restricted to the domain.
- Restrict the domain for given functions and define inverse functions of sine, cosine, and tangent, denominated $\sin^{-1}x$ or $\arcsin x$; $\cos^{-1}$ or cosine arc x, and $\tan^{-1}x$ or $\arctan x$, and trace their respective graphs.
- Establish the equivalence between symbols $f^{-1} = \arcsin (\sin^{-1}x = \text{sine arc}\, x)$, for each of the trigonometric functions.
- Establish the horizontal asymptotes for the $y = \tan^{-1}x$ function.
- Evaluate the inverse function of the sine, cosine, and tangent with the calculator.

- Establish the restriction conditions in the domain for which the inverse functions $\cot^{-1}x$, $\sec^{-1}x$, and $\csc^{-1}x$ are defined, draw their graphs, and identify its domain and field of values.

Keywords

- cosine arc
- inverse function
- periodicity
- sine arc
- tangent arc

Unit 6. Analytic Trigonometry

In this unit, we will work with the definitions of trigonometric functions from the previous unit to derive basic trigonometric identities. They are useful to establish other trigonometric identities and to develop formulas of addition, difference, and multiples of angles. They are also useful in a variety of applications, to simplify expressions, and solve trigonometric equations. Also, in this unit, we will work on the application of the Law of Sines and Cosines. Lastly, we will begin discussing polar coordinates where the points on the plane are located based on a distance and an angle (r, θ). We will work with the trigonometric forms of complex numbers, where we present complex numbers as points on a plane. We will work with the application of vectors on a plane where we visualize vectors as shifts on a plane from one point to another in a certain direction. In this sense, the shift corresponds to the distance traveled and the direction is given by the angle with respect to the abscissa. This representation is analogous to the representation of points on a plane as polar coordinates.

General Objectives

- Solve trigonometric expressions by applying formulas of addition by applying formulas of sum and difference of sines and cosines.
- Solve double-angle and half-angle of the sine, cosine, and tangent problems.
- Define the Law of Sine and Law of Cosine and apply them to solve problems with right and oblique triangles.
- Define the system of polar coordinates and graph the points on the polar plane.
- Establish the relationship between polar and rectangular coordinates, and convert polar coordinates to rectangular coordinates and vice versa.
- Convert an equation in rectangular coordinates to an equation in polar coordinates and vice versa.
- Define the graphic form of a complex number in a system of coordinates made up of a real axis and an imaginary one, and graph complex numbers.
- Define a complex number as $a + bi = z$, where $|z| = \sqrt{a^2 + b^2}$.
- Establish the polar form of a complex number as $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, where the value of r is a module of z, and θ the argument.
- Write complex numbers in polar form.
- Multiply and divide polar forms of complex numbers.
- Define De Moivre's theorem for powers and roots of complex numbers in their polar forms.
- State the roots of complex numbers.
- State the magnitude of vectors $|v| = \sqrt{a^2 + b^2}$, with vertical and horizontal shifts of: $a = x_2 - x_1,\ b = y_2 - y_1$.
- Define the product of a vector on a given scale, such as: $v = \langle a_1, b_1\rangle, cv = c\langle a_1, b_1\rangle = \langle ca_1, cb_1\rangle$.
- Define the addition and subtraction of vectors, and carry out operations with vectors.
- Solve everyday vector application problems.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Analytic Trigonometry

Code: C325G0SU06L00

Unit Documents

Lesson 1. Trigonometric Identities

Code: C325G0SU06L01

Objectives

- Establish the fundamental trigonometric identities (reciprocal, Pythagorean, even and odd) and cofunctions.
- Simplify trigonometric expressions by applying fundamental identities.
- Prove a few trigonometric identities through algebraic manipulations and following rules: starting with one side of the expression, apply known identities, and convert to sines and cosines.
- Simplify trigonometric expressions through the application of identities.

Keywords

- cofunctions
- Law of Cosines
- Law of Sines
- Pythagorean
- reciprocal
- trigonometric identities

Lesson 2. Functions of the Sum and Difference of Angles

Code: C325G0SU06L02

Objectives

- Establish the formulas for sines, cosines, and tangents, as well as additions and differences of angles.
- Solve trigonometric expressions by applying the formulas for sines, cosines, tangents, and sums and differences of angles.
- Establish the formulas for sums and differences of sines and cosines of angles and solve problems of this kind.

Keywords

- cosine of a sum or of a difference
- difference of sines
- sine of a difference
- sine of a sum
- sum of sines
- sum or difference of cosines
- tangent

Lesson 3. Double-Angle and Half-Angle Functions

Code: C325G0SU06L03

Objectives

- Establish the formulas for products of sines and cosines, and solve expressions by applying these formulas.
- Establish formulas for the angle of a sine, cosine, and tangent, and solve double-angle trigonometric expressions.
- Establish formulas for the half-angle of the sine, cosine, and tangent, and solve trigonometric expressions of the mid or half-angle.

Keywords

- double-angle
- half-angle
- product of sines and cosines

Lesson 4. Formulas for the Sum and Difference of Trigonometric Functions

Code: C325G0SU06L04

Objectives

- Define the sum and difference formulas of other trigonometric functions from the sine and cosine functions.
- Solve trigonometric expressions by applying formulas of the sum and difference of sines and cosines.
- Define the reference angle to solve trigonometric equations.

Keywords

- cosecant
- cotangent
- secant
- tangent

Lesson 5. Trigonometric Equations

Code: C325G0SU06L05

Objectives

- Define trigonometric equations as equations that contain trigonometric functions.
- Solve trigonometric equations through algebraic manipulations and knowledge of the values of trigonometric functions.
- Solve trigonometric equations through factorization.
- Solve trigonometric equations by applying identities.
- Solve trigonometric equations that contain functions of multiple angles.

Keywords

- factorization
- trigonometric equation
- trigonometric identities

Lesson 6. Law of Sines and Cosines

Code: C325G0SU06L06

Objectives

- Define the Law of Sines and the Law of Cosines.
- Apply the formula of the Law of Sines to determine the measurements of angles or sides of oblique triangles in situations where we know the measurements of two angles and one side of a triangle (*SAA*), or two sides and one angle (SS*A*).
- Apply the formulas of the Law of Cosines to determine the measurements of angles or sides of oblique triangles in situations where we know the measurements of two angles and the side between them (*ASA*), or the measurements of the three sides of the triangle (*SSS*).

Keywords

- oblique angle
- Law of Cosines
- Law of Sines
- *SAA*, *SSA*, *ASA*, *SSS* relations

Lesson 7. Polar Coordinates

Code: C325G0SU06L07

Objectives

- Define the system of polar coordinates and locate the points at a directed distance of the origin: $P(r, \theta)$.
- Represent different polar coordinates for a single point.
- Establish the relation between polar coordinates and rectangular coordinates.
- Convert polar coordinates to rectangular coordinates and vice versa.
- Convert an equation, given rectangular coordinates to an equation in polar coordinates and vice versa.
- Apply the trigonometric identities to simplify polar equations.

Keywords

- polar axis
- polar coordinates
- polar equation
- pole
- rectangular coordinates

Lesson 8. Trigonometric Forms of Complex Numbers

Code: C325G0SU06L08

Objectives

- Define the graph form of a complex number in a system of coordinates made up of a real axis and an imaginary axis, and graph complex numbers.
- Define a complex number in the form $a + bi = z$, where $|z| = \sqrt{a^2 + b^2}$.
- Establish the polar form of a complex number as $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$. The value of r is the module of z and θ is the argument.
- Write complex numbers in their polar form.
- Multiply and divide polar forms of complex numbers.
- Define De Moivre's theorem for powers and roots of complex numbers in their polar forms.
- Determine the roots of complex numbers.

Keywords

- complex number
- De Moivre's theorem
- imaginary axis
- polar form of a complex number
- real axis
- square roots of complex numbers

Lesson 9. Vectors and Applications in Trigonometry

Code: C325G0SU06L09

Objectives

- Define a vector as a segment of line that has direction and that represents a shift from one point to another.
- Define the w vector by its shift from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$, as $w = \langle x_2 - x_1, y_2 - y_1\rangle$.
- Identify horizontal shift ($a = x_2 - x_1$) and vertical shift ($b = y_2 - y_1$).
- Determine the magnitude of vectors with the formula $|v| = \sqrt{a^2 + b^2}$.
- Define the sum of the vectors for $v = \langle a_1, b_1\rangle$ and $u = \langle a_2, b_2\rangle$, as: $v + u = \langle a_1 + a_2, b_1 + b_2\rangle$.
- Define the subtraction of vectors for $v = \langle a_1, b_1\rangle$ and $u = \langle a_2, b_2\rangle$, as: $v - u = \langle a_1 - a_2, b_1 - b_1\rangle$.
- Define the product of a vector for a given scalar: $v = \langle a_1, b_1\rangle$, $cv = c\langle a_1, b_1\rangle = \langle ca_1, cb_1\rangle$.

- Define a reference angle in order to trace vectors on a plane and determine their direction.
- Carry out operations with vectors.

Keywords

- angle of reference
- components
- direction
- magnitude
- scale
- shift
- vector

Unit 7. System of Equations and Inequalities

In science, as in business and economy we tend to find models or problems that involve more than one equation with two or more variables, simultaneously. In these cases, we speak of a system of linear equations. If all the equations in a system establish linear relations, the system is called a linear system. If all of the equations in the system establish linear relations, the system is known as a linear system. In this unit we will work with different techniques to solve systems of equations or linear inequality. We will limit our attention to systems with two or three variables. We will also discuss the breakdown of rational expressions into partial fractions. At the end of a unit we will work with systems of linear equations, systems of inequalities, and linear programming.

General Objectives

- Define linear equations and their solutions as the values of variables that satisfy each one of the equations of the system of equations, simultaneously.
- Define a system of consistent linear equations as one that has a solution and whose graphs (lines) intersect at one point.
- Define a system of inconsistent equations as one which does not have a solution where the lines are parallel.
- Define a consistent and dependent system as one with infinite solutions in which the lines coincide in all of their points, since the equations in the system are equivalent.
- Solve systems of linear equations using different methods (substitution or elimination).
- Define matrixes as an abbreviation of a system of linear equations in a rectangular arrangement of rows and columns.
- Establish the symbols used to write matrixes and solve systems of equations using matrixes, through different methods.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. System of Equations and Inequalities

Code: C325G0SU07L00

Unit Documents

Lesson 1. System of Linear Equations

Code: C325G0SU07L01

Objectives

- Define a system of linear equations and its solution as the values that the variables assume that satisfy each of the equations of the system.
- Define a system of linear equations that has a solution and where the lines intersect in one point as consistent.
- Define a system of linear equations that does not have a solution, where the lines are parallel as inconsistent.

- Define a system of equations with infinite solutions where lines coincide on all of their points as consistent and dependent.
- Establish methods of substitution and elimination to solve systems of linear equations and solving problems by applying these methods.

Keywords

- consistent
- consistent and dependent
- elimination method
- inconsistent
- linear equations
- substitution method

Lesson 2. Matrixes and Row Reduction

Code: C325G0SU07L02

Objectives

- Define matrixes as the abbreviated method of a system of linear equations in a rectangular arrangement of rows and columns of coefficients.
- Establish the symbols used to write matrixes and what subscripts represent.
- Establish the method of row reductions to determine the solution of the system of equations which it represents.
- Solve systems of equations using matrixes.
- Define the addition and subtraction of matrixes, and solve problems of addition and subtraction of matrixes.
- Define the product of matrixes and the conditions that are required for this operation.

Keywords

- columns
- elimination
- matrix
- reduction
- row
- substitution

Lesson 3. Determinants

Code: C325G0SU07L03

Objectives

- Establish the method to calculate determinants of square matrixes where the number of rows is the same as the number of columns.
- Expand the determinant of a matrix in regard to a row or column.
- Transform rows and columns of matrixes to calculate a determinant.

Keywords

- cofactor
- determinant

- expansion
- square matrixes
- transform

Lesson 4. Resolution of Equations Using the Cramer and Gauss-Jordan Elimination Method
Code: C325G0SU07L04

Objectives

- Define Cramer's rule for finding a single solution to a matrix system using matrix determinants.
- Solve matrixes using Cramer's rule.

Keywords

- cofactor
- Cramer's rule
- unique solution

Lesson 5. Partial Fractions
Code: C325G0SU07L05

Objectives

- Define partial fractions as the process of rewriting or breaking down a rational expression as a sum or difference of fractions with unknown numerators.
- Express partial fractions as a system of equations.
- Solve a system of equations determining the numerators of partial fractions through algebraic manipulation.

Keywords

- linear factors
- partial fractions
- quadratic factors
- rational expressions

Lesson 6. System of Nonlinear Fractions
Code: C325G0SU07L06

Objectives

- Apply the resolution, substitution, or elimination of systems of linear equation along with the graphing method in order to solve nonlinear equation systems.
- Graph systems of nonlinear equations and determine the values of the solution of the system.

Keywords

- elimination
- graphing method
- nonlinear equation
- substitution

Lesson 7. System of Inequalities

Code: C325G0SU07L07

Objectives

- Draw graphs of systems of inequalities on the same plane and distinguish between the areas of each inequality.
- Identify the area of intersection between both graphs of inequalities that represent solving a system.
- Solve a system of inequalities to determine the points that demarcate the solution region of a system of an inequality.
- State values that demarcate a region within a system of inequalities.

Keywords

- inequality

Lesson 8. Linear Programming

Code: C325G0SU07L08

Objectives

- Establish a system of linear equations with restrictions.
- Solve a system of inequalities to determine the points that demarcate a solution region in a system of inequalities.
- Determine the values that demarcate the corners of the region between lines in a system of equations.
- Evaluate the values of the corners in the objective equation and the maximum and minimum value.
- Solve problems with maximum or minimum values applied to the theory of linear programming.

Keywords

- corners
- linear programming
- maximum
- minimum
- restrictions

Unit 8. Analytic Geometry and Conic Sections

Mathematicians in Ancient Greece are known for the development and application of geometric properties of circles, ellipses, parabolas, and hyperbolas. Conic figures are made up of the intersection between planes with different inclinations and right cones. The cut made by a plane going through a cone produces a conic section, as the Greeks established. These figures are useful for the study of the path of planets and satellites in orbit, as well as the trajectory of projectiles. Also, they are very useful tools in problems related to optics and atomic physics. In this unit, we will use techniques of analytic geometry on a plane to develop equations for ellipses, hyperbolas, and parabolas, in the rectangular coordinate system as well as the polar coordinate system.

General Objectives

- Establish the equation of the circumference of a circle under different circumstances.
- Establish and distinguish the equation of the ellipse according to its orientation in regards to a plane, determine the vertices, focus, and center of an ellipse, and trace its graph.
- Define the parabola based on its vertex, focus, and directrix, and establish the standard equation of the parabola and its directrix, depending on the orientation in regards with the plane.
- State the standard equation of a parabola given its focus, vertex, and axis of symmetry, and determine the focus and directrix of a parabola based on its equation.
- Solve problems of application in engineering with the use of parabolas.
- Establish the standard equation of the hyperbolas according to its orientation on the plane.
- State the equation that describes the asymptote of hyperbolas, based on diagonals in the central rectangle.
- State the equation of a hyperbola given its vertices and focus, or from its vertices and asymptotes.
- Define and identify the conjugate axis of a hyperbola.
- Define the translations of conic sections, establish the formulas for translations of the axes, and determine the center, foci, and vertices of conics that have suffered translations.
- Establish the general equation of any conic:
  $Ax^2 + Cy^2 + Dx + Ey + F = 0.$
- State the base of the discriminant if an equation can be an ellipse or a circle (for discriminant < 0), a parabola (discriminant $= 0$), or a hyperbola (discriminant > 0).
- Define the rotation of conic sections in a particular angle, α, and establish the formulas for rotation of the axes.
- Establish parametric equations as a general method to describe any curve where it represents a trajectory and the points on the curve are in a time function.
- Determine parametric equations for a given curve and trace the graph, substituting the values of t in each of the equations that describe the coordinates (x, y).

- Represent parametric equations in a single equation, through the elimination of the parameter, applying trigonometric identities.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Analytic Geometry and Conic Sections

Code: C325G0SU08L00

Unit Documents

Lesson 1. Circles

Code: C325G0SU08L01

Objectives

- Determine the equation for the circumference of a circle given its center and the equation of the line that is a tangent to it.
- Determine the general circumference of a circle, given three points in its circumference and by establishing a linear system of equations for the three points: $x^2 + y^2 + Ax + By + C = 0$.

Keywords

- center
- circle
- equation of the circumference

Lesson 2. Ellipses

Code: C325G0SU08L02

Objectives

- Define an ellipse and the relation between the points in a curve, vertices, and foci inside it.
- Determine and distinguish between the equations of horizontal and vertical ellipses.
- Distinguish whether a given equation represents an ellipse, establish what type of ellipse it is (horizontal or vertical), and determine its foci.
- State the vertices, foci, and center of the ellipse, and draw its graph.

Keywords

- center
- ellipse
- foci
- horizontal
- vertical

Lesson 3. Parabolas

Code: C325G0SU08L03

Objectives

- Define a parabola based on its vertex, focus, axis of symmetry, and directrix.
- Establish the standard equation of the parabola and its directrix, depending the type of parabola, be it horizontal (opening to the right or left) or vertical (opening upward or downward).
- Define the focal diameter of the parabola and its symmetry.
- Determine whether a given equation represents a parabola.
- Determine the standard equation of a parabola given its focus, vertex, and its axis of symmetry.
- Determine the focus and directrix of a parabola from its equation.
- Graph parabolas.
- Solve problems of application in engineering using parabolas.

Keywords

- axis of symmetry
- directrix
- focal diameter
- focus
- parabola
- vertex

Lesson 4. Hyperbolas

Code: C325G0SU08L04

Objectives

- Define hyperbolas in terms of their foci, vertices, axes of symmetry, transversal axes, and asymptotes.
- Establish the standard equation of hyperbolas depending on whether their transversal axis is vertical or horizontal.
- Define and identify the conjugate axis in a hyperbola.
- Determine the equation that describes asymptotes in hyperbolas, based on diagonals in the central rectangle of the hyperbola.
- Determine the equation of a hyperbola given its vertices and its foci.
- Determine the equation of a hyperbola based on its vertices and asymptotes.

Keywords

- asymptote
- axes
- central rectangle
- conjugate axis
- foci
- hyperbola
- symmetry
- vertex

Unit 9. Sequences, Series, and Mathematical Induction

We will conclude the study of graphs and functions with a brief discussion of sequences and series. A sequence is a list of numbers written in a specific order. Numbers in a sequence generally have a relation that can be established as a pattern and can also be summarized as a formula. Sequences describe events in the world. If the numbers in a sequence establish constant differences between a term and its predecessor, the sequence is arithmetic. If the relation between a term and its predecessor shows a constant rate, the sequence is geometric. As part of this unit, we will also discuss summation notation, mathematic induction, factorial notation, and the binomial theorem.

General Objectives

- Define and distinguish mathematical and geometric sequences.
- Determine the terms in arithmetic sequences from the formula that describes the sequence.
- Determine the terms in a geometric sequence based on the formula that describes the sequence and determine the common ratio r, associated with geometric progression.
- Determine a specific number in a sequence, given the formula or the sequence of numbers, and determine a certain quantity of terms in sequences given certain conditions.
- Define the symbol Σ (sigma), evaluate partial summations, and define and apply the basic properties of the summations when you determine a given summation.
- Apply the principles of summation to financial problems.
- Define the principle of mathematical induction and establish the steps to show a supposition using this principle.
- Demonstrate whether suppositions of sequences are valid for all natural numbers using the principle of mathematical induction.
- Demonstrate the utility of the principle of mathematical induction to develop the binomial theorem $(a+b)^n$ using the pattern in Pascal's triangle.
- Apply the binomial theorem to develop small values for n using Pascal's triangle.
- Define factorial notation $n! = n(n-1)(n-2)(n-3)\cdots 2\cdot 1$ and establish that $0! = 1$.
- Define the binomial coefficient as a combinatorial formula and define the binomial theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$.

At the end of this unit, students will fulfill the following objectives as developed over several lessons.

Lesson 0. Sequences, Series, and Mathematical Induction

Code: C325G0SU09L00

Unit Documents

Lesson 1. Sequences

Code: C325G0SU09L01

Objectives

- Define infinite sequences as functions with domains that are a set of whole positive numbers, where each value in the sequence denominates terms in the sequence: $a_1, a_2, a_{3,}, \dots, a_n, \dots$
- Define and distinguish between arithmetic and geometric sequences.
- Establish whether a sequence of numbers is an arithmetic sequence, finding the difference between its terms or if the sequence is geometric, determining the common ratio r, associated with geometric progression.
- Determine a specific term in a sequence, given the formula of sequence of numbers.
- Classify numerical sequences as arithmetic, geometric, or neither.

Keywords

- arithmetic sequence
- common difference
- common ratio
- geometric sequence
- sequence or series

Lesson 2. Summation and Series

Code: C325G0SU09L02

Objectives

- Classify numerical sequences as arithmetic, geometric, or neither.
- Define the symbol Σ (sigma) as notation that summarizes a summation of terms and translate expressions with Σ (sigma) when evaluating partial summations of sequences.
- Define and apply the basic properties of summations when determining a given summation.
- Establish the formula for the specific summation of a certain quantity of terms in an arithmetic sequence (partial sums).
- Establish the formula for the specific summation of a certain quantity of terms in a geometric sequence.
- Evaluate and simplify summations by applying the related properties.
- Determine a certain quantity of terms in sequences, given certain conditions.
- Apply the principles of summation to financial problems.

Keywords

- arithmetic sequence
- geometric sequence
- partial sums
- sigma
- summation
- terms

Lesson 3. Mathematical Induction

Code: C325G0SU09L03

Objectives

- Define the principle of mathematical induction as a method for demonstrating whether statements (suppositions) about numerical sequences are true for all whole positive numbers.
- Establish the steps for proving a supposition using the mathematical induction method.
- Prove whether suppositions about sequences are valid for all natural numerals using the principle of mathematical induction.

Keywords

- mathematical induction
- suppositions

Lesson 4. Binomial Theorem

Code: C325G0SU09L04

Objectives

- Prove the utility of the principle of mathematical induction to develop the binomial theorem $(a+b)^n$, with the pattern that contains Pascal's triangle.
- Define the binomial theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$.
- Apply the binomial theorem to develop small values for n, using Pascal's triangle.
- Define the factorial notation $n! = n(n-1)(n-2)(n-3) \cdots 2 \cdot 1$, and establish that $0! = 1$.
- Define the coefficient of a binomial as a formula of the combinatorial formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, having n and k be whole numbers, so that $0 \leq k \leq n$.

Keywords

- binomial theorem
- combinatorial notation
- factorial notation
- Pascal's triangle