edusystem by Dreyfous

DREYFOUS

Course Overview

Trigonometry

DREYFOUS

TABLE OF CONTENTS

Course Overview 1

Course Framework 3

Unit Breakdown 8

Unit 1. Fundamentals of Trigonometry 8

Unit 2. Trigonometric Functions of Any Angle 12

Unit 3. Variations of Trigonometric Ratios and Their Graphs 15

Unit 4. Relationship Between the Trigonometric Functions of an Angle and Trigonometric Identities 18

Unit 5. Trigonometric Functions of the Sum and Difference of Two Angles 21

Unit 6. Multiple and Submultiple Angles 23

Unit 7. Formula to Transform the Sum or Difference of the Functions of Two Angles into a Product 25

Unit 8. Solving Right Triangles and Applications 27

Unit 9. Solving Non-Right Triangles and Applications 29

Unit 10. Area of Polygons 31

Unit 11. Graphs, Polar Equations, and Vectors 33

Course Overview

The objective of EduSystem's Trigonometry course is to develop a high level of mathematical skills in the student and to create awareness about the importance of the study of trigonometry to address real-world problems and situations. Through the developed content and the strategies and techniques used, we encourage the student to gain a deep understanding of the concepts, as well as the technical skills necessary for the study of later courses on calculus and its applications. The way the topics, examples, and applications used are introduced and presented, as well as the way math skills are developed, allow the student to visualize, understand, and value their usefulness in everyday life. For this purpose, we introduce trigonometric applications in the fields of engineering, land surveying, wave motion, vibrations, sound, alternating currents, and thermodynamics, among others.

The areas and topics to be discussed in this course include: fundamentals of trigonometry; trigonometric ratios of any angle; variations of trigonometric ratios and their graphs; relationship between an angle's trigonometric ratios and their focus as functions; trigonometric identities; formulas for the sum or difference of two angles; trigonometric functions of angles and their multiples; transformation of the sum or difference of the functions of two angles into a product; solving right and non-right triangles and their applications; the area of polygons; and graphs of polar equations and vectors with their applications.

This content takes into account the *Core Standards of the Department of Education of Puerto Rico* (Puerto Rico Core Standards, 2014) and the *Common Core State Standards* of the United States. The outline of objectives per lesson extensively considers all the skills and concepts necessary for the student to make the connections between the various standards (number and operations, algebra, functions, geometry, measurement, and data analysis) into which mathematics are currently categorized and which are intertwined in the Trigonometry course. The teaching method focuses on conceptual understanding, skill development, and mathematical problem solving, along with the development of critical thinking skills, as a means for the integral formation of the student.

The course deliberately integrates content related to science, technology, and engineering, among others, with a dual purpose: to encourage students to see the direct

application of what they learn and to visualize the importance of mathematics as a universal discipline at the service of society and its institutions. On the other hand, the incorporation of everyday life situations and problems in each of the topics discussed aims to awaken the student's interest in the study of the discipline.

Course Framework

The Trigonometry course consists of eleven units, carefully structured into different lessons. The number of lessons per unit depends on the scope and depth with which the different topics are discussed and developed. Each presentation includes conceptual definitions, demonstrations, structures, concrete examples, explanations, multiple representations, and practice exercises, as well as the application of concepts and skills in everyday life situations. The lessons also include practice exercises, quizzes, additional practice laboratories, assignments, self-assessment exercises, and a descriptive log detailing information for the teacher in order to facilitate their daily planning.

Some lessons also include laboratories that introduce and reinforce algebra and geometry concepts that are directly linked to trigonometry. The lessons also incorporate the use of technological tools, such as calculators and programs to draw and create shapes and triangles. Additionally, these lessons feature research activities in order to pique the curiosity and the particular interests of each student. The practice and self-assessment activities seek to make students aware of their strengths and weaknesses in terms of the mastery of content, with the purpose of allowing them to gradually take control of their learning. The teacher, as an integral and essential part of the process, will have the responsibility of stimulating, orienting, guiding, and periodically evaluating each of the students' acquired knowledge.

Units are made up of the following sections:

Lessons

Each unit is made up of different lessons, divided into topics, macro concepts, and skills. In turn, each lesson consists of five key elements: presentation or course content, digital documents (PDF), internet links, self-assessment, and descriptive log.

- **Descriptive Log.** This is the detailed lesson plan. It includes the lesson's specific objectives, standards, and expectations, teaching strategies and learning resources, keywords, internet links, references, among others. The only person with access to the descriptive logs will be the teacher.

- **Lesson Content.** Each presentation contains detailed explanations of the lesson's concepts and skills, as established in the objectives. In addition, it contains the following elements, which systematically contribute to the student's expected learning development:
  - **Examples.** In each section, when developing skills, we include examples that explain the solution to an exercise or problem step-by-step, so that the student reviews the concepts and skills presented.

  - **Practice**. This includes a series of carefully selected exercises in order for the student to practice the skills and concepts discussed. The activity is designed to periodically assess the learning progress of students before moving on to other topics and skills. It does not include procedures or explanations; it only includes the solution to the exercises.

  - **Solution.** It is used to hide the solution to an exercise or problem that the student should try to answer. Once you click on this icon, the solution or answer to the exercise will be displayed.

  - **Procedure.** This icon shows the steps or the algorithm to follow in order to solve an exercise or a problem.

  - **Demonstration.** It presents formal demonstrations of theorems or the derivation of important formulas or algorithms.

- **Calculator.** It includes the explanation of processes when using and handling the calculator to solve the section's exercises. It also connects the student to the virtual graphing calculator.

- **Animation.** It gives access to explanations, procedures, or graphics that provide a visual portrayal of the concepts and skills discussed in the section. Animations support the student's conceptual development and understanding.

- **Definition.** It includes formal mathematical definitions of concepts and processes mentioned or discussed. It also includes axioms and theorems that are important for the lesson.

- **Biography.** It includes a short biography of the mathematician or scientist credited with developing the definition, formula, procedure, or demonstration used in the lesson.

- **Note.** This icon points out common errors or reinforces details that should not be forgotten.

- **Did You Know...** This icon accesses a section showing an explanation or situation that connects aspects of everyday life with the mathematical skills and concepts discussed. In some cases, this section shows the link between the development of logical thinking in humans with certain mathematical skills and processes.

- **Tabs.** These can be found on the right or left side of the presentation. They unfold to the left and may include different elements, such as flowcharts, biographies, notes, photos, explanations, suggestions, reminders, "Did You Know...", or necessary background knowledge.

- **Incorrect.** It states when the student has selected an incorrect answer in the practice exercises.

- **Correct.** It indicates the correct selection to answer an exercise or practice problem.

- **Photograph or image.** A particular explanation is connected to a photo or image, which will be likely accessed by internet.

- **Video.** It provides access to a short video that links the mathematical content to everyday life.

- **Internet.** It is a direct link to a web site or portal closely related to the topic.

Each section included in the presentation is connected to a particular identifying icon, as shown in the explanation provided above. The initial course presentations include an icon with the word that describes the section. This way, the student will start becoming familiar with what each icon represents. Later presentations only include the icon that gives access to the section. Clicking on the icon will take the presentation immediately to the specific section it represents.

- **PDF Documents.** These documents include a copy of practice exercises from the lesson, an additional practice section, activities that require the use of a calculator, or assignments. These documents may be printed out for students to work on. Assignments are exercises and problems that the students work on at home, and that allow them to practice in order to strengthen the skills and concepts learned. These assignments are optional and may be used at the teacher's discretion.
- **Internet Links.** These links are a direct connection to the internet and may be accessed directly from the presentation. They include additional explanations, examples, applications, or demonstrations that facilitate the students' conceptual development of the skills and topics discussed.
- **Self-Assessment.** It consists of practice tests that the student answers to monitor their own learning before taking the formal unit evaluation tests offered by the teacher.

Unit Documents

- **Lesson 0.** This is not a content lesson but rather a file containing the unit elements: PDF documents, unit evaluations, and an introduction video, among others.
    - **PDF Documents.** Documents containing laboratory activities for application and conceptual development, laboratory activities for skill development, calculator activities, additional exercises, and assessment activities.
    - **Evaluations.** It includes the evaluations to be used in the unit: unit pretest and posttest. Lesson 00 of Unit I includes a diagnostic test for the entire course.
    - **Links.** It includes internet links that may be solely for the teacher's use and which they may share with the students.

Unit Breakdown

Below, we have outlined the course content with its respective units, lessons, general objectives per unit, unit and lesson topics or titles, as well as the specific objectives and concepts per lesson.

General Course Objectives

- Promote the development of spatial and geometric thought by teaching how to solve triangles, through basic to more complex exercises.
- Build, gradually and significantly, concepts and skills related to the Pythagorean theorems, trigonometric ratios, sine and cosine laws, and their relationship with application to determine the elements of a triangle, given different situations.
- Encourage conducive learning environments that engage and motivate students to explore the knowledge on topics related to trigonometry, as well as its use for solving triangles with applications to real-life problems using pencil, paper, ruler, protractor and compass, and technological tools.

Unit 1. Fundamentals of Trigonometry

This unit introduces trigonometry as an area of math focused on triangles and the relationship between the sides and angles of right triangles. Some history is provided for this discipline, starting with ancient civilizations like Greece and the Egypt where the first applications of trigonometry addressed astronomy and navigation. The trigonometric ratios for right triangles are established, as well as the trigonometric values for special angles.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Underline the importance of trigonometry, within the historical framework of human beings, for the development of mathematical-geometric knowledge.
- Establish the various trigonometric ratios for a right triangle and the relationships between them.
- Evaluate trigonometric ratios.
- Determine the trigonometric ratios of an angle based on the measurements of a right triangle.
- Express one trigonometric ratio as a function of another.
- Apply the trigonometric ratios to everyday problems.
- Determine the trigonometric values for the following special angles: 30°, 45°, and 60°.

Lesson 1. Historical Data, Direct and Indirect Measurements

Code: C314G0SU01L01

Objectives

- Become familiarized with historical facts from ancient Greece and Egypt, as precursors to trigonometry as a field of study, where triangles and their measurements were first studied for construction, as well as for astronomy and navigation purposes.
- Identify the contributions of different mathematicians throughout time, such as Thales of Miletus, Hipparchus of Nicaea, Claudius Ptolemy, Aristarchus of Samos, Bartholomaeus Pitiscus, François Viète, John Napier, and Leonhard Euler.
- Review concepts related to triangles, such as *the Pythagorean theorem* and the *sum of the interior angles of a triangle*, as well as related elements in the coordinates plane, such as a circumference equation and the distance between two points.
- Define and determine indirect measurements by using scales, ratios, and proportions.
- Establish a formal definition for trigonometry as a science that studies the metric relationships in triangles.

Concepts

- direct measurements
- indirect measurements
- proportions
- ratios
- trigonometry

Lesson 2. Trigonometric Functions of Acute Angles

Code: C314G0SU01L02

Objectives

- Review concepts related to triangles, such as sides, angles, triangle similarity, and ratios between triangle sides.
- Establish all possible ratios between triangle sides.
- Define the trigonometric ratios based on the ratios between the sides of a right triangle.
- Become familiarized and define the concepts of hypotenuse, legs, opposite side, and adjacent side, which are related to right triangles.
- Define and identify the trigonometric ratios $sin\ \theta$, $cos\ \theta$, and $tan\ \theta$ for a θ angle.
- Define and identify the reciprocal ratios of trigonometric ratios: $csc\ \theta$, $sec\ \theta$, and $cot\ \theta$.
- Become familiarized with the process of rationalizing the denominator in irrational expressions.

- Apply the Pythagorean theorem to determine the length of any side in a right triangle.
- Determine the value of the trigonometric ratios of triangles.
- Establish the proportionality between the sides of right triangles and determine any missing measurements.

Concepts

- legs
- cosine
- reciprocal functions
- hypotenuse
- adjacent side
- opposite side
- ratios
- trigonometric ratios
- sine
- tangent

Lesson 3. Trigonometric Functions of 30°, 45°, and 60°

Code: C314G0SU01L03

Objectives

- Demonstrate the trigonometric values of 30° and 60° angles using an equilateral triangle and the corresponding height of one of its vertices.
- Demonstrate the trigonometric values of a 45° angle using an isosceles right triangle.
- Determine the value of trigonometric expressions containing 30°, 45°, and 60° angles.

Concepts

- coterminal angles
- angle in standard position
- equilateral triangle
- isosceles triangle
- trigonometric values

Lesson 4. Functions of Complementary Angles

Code: C314G0SU01L04

Objectives

- Establish the trigonometric relationships of complementary angles.
- Use complementary and supplementary angle identities to determine the value of trigonometric expressions.

Concepts

- complementary angles

- complementary functions

Unit 2. Trigonometric Functions of Any Angle

This unit focuses on working on angles on the coordinates plane based on the trigonometric relationships developed in the unit circle and the Pythagorean theorem. We will determine measurements of angles whose terminal side contains points on the coordinates plane. We will also determine the trigonometric values for quadrant angles and for angles equal to 30^0, 45^0, 60^0, and 90^0. In addition, we will also convert angle measurements from the sexagesimal system to radians and vice versa.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Determine the trigonometric values for angles formed in any quadrant of the coordinates plane.
- Identify positive angles, negative angles, and angles of more than 360°.
- Determine the trigonometric values for angles formed on the coordinates plane.
- Convert angle measurements from the sexagesimal system into radians and from radians into the sexagesimal system.

Lesson 1. Generating Angles and Rectangular Coordinates

Code: C314G0SU02L01

Objectives

- Determine the equivalent angle and the reference angle formed by a terminal side containing an ordered pair on the coordinates plane.
- Determine the ratios of an angle based on a given ordered pair on the coordinates plane.
- Define the corresponding sign of trigonometric ratios based on their quadrant on the coordinates plane.

Concepts

- reference angle
- rectangular coordinates
- generation of angles
- terminal side
- coordinates plane

Lesson 2. Trigonometric Functions of Angles on Different Quadrants and the Unit Circle
Code: C314G0SU02L02

Objectives

- Define the unit circle.
- Determine the trigonometric ratios of points on the unit circle.
- Determine equivalent angles formed by a terminal side containing a point on the unit circle and the x-axis.
- Determine trigonometric values for any angle multiple of 30^0, 45^0, and 60^0.
- Determine trigonometric values for angles formed on (quadrants) axes such as 90^0, 180^0, 270^0, and 360^0.

Concepts

- quadrant angles
- reference angle
- equivalent angles
- unit circle
- coterminal side

Lesson 3. Reducing Angles in the Second, Third, or Fourth Quadrant to the First Quadrant
Code: C314G0SU02L03

Objectives

- Simplify angles greater than $360°$ and negative angles, and determine their equivalent between $0°$ and $360°$.
- Define supplementary angles, such as angles that add up to 180^o.
- Establish the relationship between an angle in the second quadrant and an angle in the first quadrant based on their difference with 180^0.
- Establish the relationship between the trigonometric ratios of an angle in another quadrant with the trigonometric ratios of a reference angle in the first quadrant in order to reduce angles in the second, third, or fourth quadrant to the first quadrant: quadrant II $(180^o - \alpha)$, quadrant III $(180^o + \alpha)$, and quadrant IV $(360^o - \alpha)$.
- Solve problems for positive angles of less than one turn, where the sign is given by the established trigonometric ratio and the quadrant to which said angle belongs.
- Solve problems for positive angles of more than one turn, where the angle is divided by 360^0 as we work with the remainder.

Concepts

- reference angle
- angles greater than 180^0
- supplementary angles
- angle reduction

Lesson 4. The Sexagesimal System and Radians

Code: C314G0SU02L04

Objectives

- Establish the equivalences that 1 degree equals 60 minutes ($1^0 = 60'$) and 60 seconds equal 1 minute ($60'' = 1'$), and express degrees in minutes and seconds and vice versa.
- Define a radian as the measure of an angle where the arc length is equal to the radius.
- Establish the factor or formula to convert degrees into radians ($\alpha^0 \cdot \frac{\pi}{180^0}$) and perform angle conversions from degrees into radians.
- Establish the formula or factor to convert radians into degrees ($x\pi \cdot \frac{180^0}{\pi}$) and convert radians into degrees.
- Determine the trigonometric ratios of angles given in radians.

Concepts

- degrees
- minutes
- radians
- seconds
- sexagesimal system

Unit 3. Variations of Trigonometric Ratios and Their Graphs

This unit starts illustrating trigonometric ratios as functions. These functions depend on the size of the angle formed between the x-axis as the initial side and a line or ray passing through the origin on the plane as the terminal side of the angle. The trigonometric ratios on the coordinates plane are defined as functions and their values are defined based on the angle or the triangle sides. The unit will also illustrate the periodic properties of the sine, cosine, and tangent ratios, as well as their applications to everyday situations, such as simple harmonic motion.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Define the trigonometric ratios stemming from the unit circle, using a point $P(x, y)$ on the circumference determined by t.
- Illustrate trigonometric ratios as functions.
- Associate the coordinates of the points on the unit circle with values within the set of real numbers, determined by a distance t (arc of the circumference).
- Define the domain and range of trigonometric functions.
- Graph trigonometric functions.
- Apply trigonometric functions to everyday problems exhibiting periodicity.
- Establish and determine the amplitude and period of sine and cosine functions.

Lesson 1. Geometric Representation of Trigonometric Ratios

Code: C314G0SU03L01

Objectives

- Establish the periodic properties of the sine and cosine ratios: period, amplitude, phase shift.
- Define the sine and cosine ratios as a function and graph them establishing their period and amplitude.
- Define the graphs of the sine and cosine functions as a waveform called simple harmonic motion.
- Establish the characteristics of simple harmonic motion and determine the amplitude, angular frequency, and phase angle in these functions.
- Define the term "oscillation frequency" and establish its formula.
- Determine the oscillation frequency given the conditions of a harmonic motion.

Concepts

- range
- amplitude
- phase angle

- phase shift
- angular frequency
- oscillation frequency
- function
- harmonic motion
- period

Lesson 2. Transformations of the Sine, Cosine, and Tangent Functions

Code: C314G0SU03L02

Objectives

- Define the graphs of the sine, cosine, and tangent functions for an angle.
- Determine the amplitude and the period for the trigonometric functions graph.
- Establish the asymptote corresponding to the graph of the tangent function of an angle.
- Explain the graphs of other trigonometric functions at different intervals in the domain and the relationship with their reciprocal function.
- Determine which of the sine, cosine, and tangent functions are even or odd based on their graphs.
- Establish the periodic properties of the sine and cosine functions: period, amplitude, phase shift.
- Define the graphs of the sine and cosine functions as a waveform called simple harmonic motion.
- Establish the characteristics of simple harmonic motion and determine the amplitude, angular frequency, and phase angle in these functions.
- Define the term "oscillation frequency" and establish its formula.
- Determine the oscillation frequency given the conditions of a harmonic motion.

Concepts

- range
- amplitude
- phase angle
- asymptote
- phase shift
- angular frequency
- oscillation frequency
- odd function
- even function
- graph
- harmonic motion
- period

Lesson 3. Transformations of the Secant, Cosecant, and Cotangent Functions

Code: C314G0SU03L03

Objectives

- Define the graphs of cotangents, secants, and cosecants based on the sine and cosine functions.
- Establish the vertical asymptotes for each one.
- Explain the graphs of other trigonometric functions at different intervals in the domain and the relationship with their reciprocal function.
- Determine which of the cotangent, secant, or cosecant functions are even or odd based on their graphs.

Concepts

- amplitude
- asymptote
- odd function
- even function
- reciprocal function
- graph
- period

Unit 4. Relationship Between the Trigonometric Functions of an Angle and Trigonometric Identities

This lesson will highlight trigonometric identities, from their definition to demonstrations and applications for different angle values. We will solve exercises to determine the exact angles of trigonometric expressions in degrees or radians, with and without the use of a calculator.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Establish that an identity is represented by a true equation for any value of the variable for which both sides of the equation are defined.
- Define and demonstrate trigonometric identities:
  - $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- Define and demonstrate the Pythagorean identities of an angle in a standard position with a terminal side intersecting the unit circle at point (x, y).
  - $\cos^2\theta + \sin^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$
- Define and demonstrate even and odd identities related to the trigonometric functions of angle θ and $-\theta$: a) $\sin(-\theta) = -\sin\theta$; b) $\cos(-\theta) = \cos\theta$; c) $\tan(-\theta) = -\tan\theta$; d) $\cot(-\theta) = -\cot\theta$; e) $\sec(-\theta) = \sec\theta$, and f) $\csc(-\theta) = -\csc\theta$.
- Solve exercises by applying fundamental trigonometric identities using a calculator.
- Simplify trigonometric expressions using fundamental identities.
- Determine exact values of trigonometric expressions of angles in degrees and radians, using even and odd identities of trigonometric functions, without a calculator.

Lesson 1. Relationships Between Trigonometric Functions

Code: C314G0SU04L01

Objectives

- Demonstrate trigonometric identities based on the Pythagorean theorem.
- Establish that an identity is represented by an equation that is true for any value of the variable for which both sides of the equation are defined.
- Define and demonstrate the trigonometric quotient identities, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$.
- Define and demonstrate the Pythagorean identities for an angle in standard position with point (x, y),
  - $\cos^2\theta + \sin^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$

Concepts

- angle in standard position
- true equation
- quotient identity
- fundamental identity
- trigonometric identity
- Pythagorean theorem

Lesson 2. Representation of Trigonometric Ratios and Identities

Code: C314G0SU04L02

Objectives

- Express all trigonometric ratios as a function of the sine of an angle.
- Express all trigonometric ratios as a cosine function.
- Express all trigonometric ratios as a tangent function.
- Solve trigonometric ratio representation problems as a function of other ratios.

Concepts

- asymptotes
- cosecant
- cotangent
- reciprocal function
- fundamental identity
- trigonometric identity
- secant
- tangent
- Pythagorean theorem

Lesson 3. Trigonometric Identities (Part 1)

Code: C314G0SU04L03

Objectives

- Simplify trigonometric expressions using fundamental identities.
- Determine exact values for trigonometric expressions of angles in degrees and radians, using the even and odd identities of the trigonometric functions, without a calculator.
- Use trigonometric identities to simplify and evaluate trigonometric expressions.

Concepts

- evaluating trigonometric expressions
- trigonometric identities
- simplify

Lesson 4. Trigonometric Identities (Part 2)

Code: C314G0SU04L04

Objectives

- Simplify trigonometric expressions using fundamental identities.
- Determine exact values for trigonometric expressions of angles in degrees and radians, using the even and odd identities of the trigonometric functions, without a calculator.
- Use trigonometric identities to simplify and evaluate trigonometric expressions.

Concepts

- evaluating trigonometric expressions
- trigonometric identities
- simplify

Unit 5. Trigonometric Functions of the Sum and Difference of Two Angles

In this unit, different angles will be expressed in terms of the sum or the difference of special angles of which we know their trigonometric ratios. This representation of angles as addition or subtraction of other angles will be used to develop the sine and cosine formulas for the sum or difference of two angles. Using the trigonometric identities and the sine and cosine formulas for the sum and difference of angles, the student will develop the formulas for all other trigonometric functions.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Determine the sine and cosine of the sum or difference of two angles.
- Develop the formula for the tangent and cotangent of the sum or difference of two angles.
- Determine the tangent and cotangent of the sum or difference of two angles.

Lesson 1. Sine and Cosine of the Sum or Difference of Two Angles

Code: C314G0SU05L01

Objectives

- Demonstrate the sine and cosine formulas for the sum or difference of two angles.
- Use the sine and cosine formulas for the sum of two angles to simplify trigonometric expressions.
- Decompose an angle by adding and subtracting known angles.

Concepts

- cosine of a sum or a difference
- difference of angles
- difference of sines
- sine of a sum
- sum of angles

Lesson 2. Tangent of the Sum or Difference of Two Angles

Code: C314G0SU05L02

Objectives

- Develop the tangent and cotangent formulas for the sum of two angles based on the sine and cosine formulas for the sum of two angles.
- Determine the trigonometric values of the sum or difference of two known angles.
- Demonstrate trigonometric identities using the formulas of the sum or difference of two angles.

Concepts

- cotangent
- cotangent of a sum
- difference of cotangent
- tangent
- tangent of a difference
- tangent of a sum

Lesson 3. Secant, Cosecant, and Cotangent of the Sum or Difference of Two Angles

Code: C314G0SU05L03

Objectives

- Develop the secant, cosecant, and cotangent formulas for the sum of two angles based on the sine and cosine formulas for the sum of two angles.
- Determine the trigonometric values of the sum or difference of two known angles.
- Demonstrate trigonometric identities using the formulas of the sum or difference of two angles.

Concepts

- cosecant of the sum or difference of angles
- cotangent of the sum or difference of angles
- secant of the sum or difference of angles

Unit 6. Multiple and Submultiple Angles

Using the trigonometric identities and the representation of angles in terms of multiples or halves of special angles, we can determine the trigonometric values of different angles. In this lesson we will define the trigonometric functions for multiples and halves of special angles.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Develop the formulas for the trigonometric functions of a double angle.
- Determine the trigonometric functions of a double angle.
- Develop the formulas for the trigonometric functions of a half angle.
- Determine the values of the trigonometric functions of a half angle.

Lesson 1. Sine, Cosine, and Tangent of a Double Angle

Code: C314G0SU06L01

Objectives

- Develop the sine and cosine formulas for a double angle.
- Determine the values of the trigonometric functions of a double angle.

Concepts

- double angle
- trigonometric identities

Lesson 2. Secant, Cosecant, and Cotangent of a Double Angle

Code: C314G0SU06L02

Objectives

- Develop the tangent and cotangent formulas of a double angle.
- Develop the secant and cosecant formulas of a double angle.
- Determine the values of the trigonometric functions of a double angle.

Concepts

- double angle
- trigonometric identities

Lesson 3. Sine, Cosine, and Tangent of a Half Angle

Code: C314G0SU06L03

Objectives

- Develop the sine, cosine, and tangent formulas of a half angle.
- Demonstrate the trigonometric identities based on the basic identities of a half angle.

Concepts

- trigonometric identities
- half angle

Lesson 4. Secant, Cosecant, and Cotangent of a Half Angle

Code: C314G0SU06L04

Objectives

- Develop the secant, cosecant, and cotangent formulas of a half angle.
- Demonstrate the trigonometric identities based on the basic identities of a half angle.

Concepts

- trigonometric identities
- half angle

Lesson 5. Trigonometric Functions of a Triple Angle

Code: C314G0SU06L05

Objectives

- Develop the formulas for the trigonometric functions of a triple angle.
- Demonstrate trigonometric identities based on the identities related to a triple angle.

Concepts

- trigonometric identities
- triple angle

Unit 7. Formula to Transform the Sum or Difference of the Functions of Two Angles into a Product

This lesson covers many applications of algebraic thinking to trigonometry. We will use the acquired knowledge of trigonometric identities and the formulas developed for sums and differences, multiple angles, and half angles to simplify trigonometric expressions.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Apply algebraic knowledge to simplify trigonometric expressions.
- Develop formulas to transform the sum and difference of angles into a product.
- Demonstrate trigonometric identities using the formulas previously learned.

Lesson 1. Transform the Sum or Difference of the Sines and Cosines of Two Angles into a Product

Code: C314G0SU07L01

Objectives

- Simplify trigonometric expressions using the formulas to transform the sum or difference of the sine and cosine functions of two angles into a product.
- Use the identities of complementary angles to simplify expressions of the sum or difference of two different trigonometric functions.

Concepts

- product of angle functions
- sum or difference of angle functions
- transformation

Lesson 2. Transform a Product into a Sum or a Difference

Code: C314G0SU07L02

Objectives

- Simplify trigonometric expressions using the formulas to transform a product into a sum or difference of the sine and cosine functions of two angles.
- Demonstrate the formulas to transform a product into a sum or difference of two trigonometric functions.

Concepts

- transformation
- product of angle functions

Lesson 3. Solving Trigonometric Equations

Code: C314G0SU07L03

Objectives

- Define the concept of trigonometric equations as those equations containing trigonometric functions.
- Solve trigonometric equations through algebraic manipulations and the knowledge of the values of trigonometric functions.
- Solve trigonometric equations using factoring.
- Solve trigonometric equations using identities.
- Solve trigonometric equations containing functions of angle multiples.

Concepts

- trigonometric equation
- inverse function

Unit 8. Solving Right Triangles and Applications

The specific purpose of this unit is to determine the measurement of the angles or the side lengths of different triangles by applying the knowledge acquired in previous lessons to solve trigonometric equations. It is also expected that the student can work with everyday applications of the knowledge acquired so far.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Determine the measurements of all the elements of a right triangle.
- Solve a right triangle with the given measurements of different elements in the triangles.
- Develop the formulas to calculate the area of a right triangle.

Lesson 1. Solving Right Triangles with Two Known Legs

Code: C314G0SU08L01

Objectives

- Using right triangles and trigonometric functions, determine the measurement of all the elements.
- Solve a right triangle based on the measurements of its legs.
- Solve application problems using trigonometric functions in right triangles.

Concepts

- right triangle
- solving triangles

Lesson 2. Solving Right Triangles with a Known Hypotenuse and a Leg

Code: C314G0SU08L02

Objectives

- Solve right triangles using the measurement of the hypotenuse and a leg.
- Solve verbal problems by solving right triangles.

Concepts

- right triangle
- solving triangles

Lesson 3. Solving Right Triangles with a Known Acute Angle and a Side

Code: C314G0SU08L03

Objectives

- Solve right triangles using the measurement of an angle and a side.
- Solve verbal problems by solving right triangles.

Concepts

- solving triangles
- right triangle

Lesson 4. Applications of Solving Right Triangles

Code: 314G0SU08L04

Objectives

- Solve right triangles using any combination of the triangle elements.
- Solve verbal problems by solving right triangles.

Concepts

- solving triangles
- right triangle

Unit 9. Solving Non-Right Triangles and Applications

In this unit, we will use the concepts and skills we worked on for right triangles and apply them to solve non-right or oblique triangles. This is possible since, for every triangle, the sum of its angles is 180^0, and there is a relationship between the lengths of the sides of a triangle and the measurement of the opposing angle. As we saw before, it is possible to solve triangles as long as we know three of its elements, one of them being the measurement of one of its sides. Oblique triangles include acute-angled and obtuse-angled scalene triangles, isosceles triangles that do not have a 90^0 angle, and equilateral triangles. To solve these triangles, we can apply the sine theorem to develop a formula that will help us determine the measurement of a side or an angle based on the proportionality between the sides and angles, provided that we establish the corresponding height of a given angle. Likewise, we can apply the cosine theorem and develop a formula to find the measurements of the sides in a non-right triangle.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Solve non-right or oblique triangles using the law of sine.
- Solve non-right or oblique triangles using the law of cosine.
- Solve application problems relating oblique triangles.

Lesson 1. Solving Triangles with the Law of Sines

Code: C314G0SU9L01

Objectives

- Develop the formula for the law of sines based on a non-right triangle.
- Solve non-right triangles using the law of sines.
- Apply the formulas of the law of sines to determine the measurement of the angles or sides of oblique triangles in cases where we have the measurements for either one side and two angles (SAA) of the triangle or two sides and one angle (SSA).

Concepts

- law of sines
- oblique triangles
- acute triangle
- obtuse triangle

Lesson 2. Solving Triangles with the Law of Cosines

Code: C314G0SU9L02

Objectives

- Develop the formula for the law of cosines based on a non-right triangle.
- Solve non-right triangles using the law of cosines.
- Apply the formulas for the law of cosines to determine the measurements of the angles or sides of oblique triangles in cases where we have the measurements for either two angles and the side between the angles (ASA) or the tree sides (SSS) of the triangle.

Concepts

- law of cosines
- acute triangle
- obtuse triangle

Lesson 3. Applying the Laws of Sines and Cosines

Code: C314G0SU9L03

Objectives

- Determine the number of solutions for a non-right triangle.
- Develop the area formula for triangles based on the law of sines and on the law of cosines.
- Use the laws of sines and cosines to solve practice problems.
- Use Heron's formula to find the area of triangles.

Concepts

- area of triangles with trigonometric ratios
- acute triangle
- obtuse triangle

Unit 10. Area of Polygons

In this unit, we will determine the area of different polygons, based on the areas of triangles, since all polygons may be divided into triangular polygon regions. The concepts and skills acquired in previous lessons can help us determine the different elements in any triangle as well as the area of these triangles, either by applying Heron's formula for a triangle's area when we know all other measurements or by establishing the relationship with the commonly known area formula.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Define and establish Heron's formula to determine a triangle's area, knowing the measurements of all its sides.
- Decompose a polygon's area into different non-overlapping triangular regions and determine the area of each triangle to find the total area of the polygon by adding the areas of the triangles.
- Apply the acquired trigonometric knowledge to determine the missing parts among the triangles making up the areas in different polygons.
- Determine the area of a polygon by applying the trigonometric knowledge acquired.
- Solve everyday situations where it is necessary to know the areas of polygonal shapes.

Lesson 1. Area of Quadrilaterals

Code: C314G0SU10L01

Objectives

- Decompose the area of a quadrilateral into triangular areas or regions.
- Determine the parts or elements in the triangles that make up a quadrilateral by using trigonometry.
- Determine the areas of triangles by applying Heron's formula.
- Determine the areas of different triangular regions in a quadrilateral by applying trigonometric knowledge (laws of sines or cosines).
- Apply the common area formula $A_{\Delta} = \frac{1}{2} \cdot b \cdot h$ and the figure of a right triangle to establish that $A_{\Delta} = \frac{1}{2} \cdot b \cdot c \cdot \sin A$.

Concepts

- triangular area
- decomposition
- Heron's formula

Lesson 2. Area of Polygons and Applications

Code: C314G0SU10L02

Objectives

- Decompose the area of different polygons into triangular areas or regions.
- Determine the parts or elements in the triangles that make up these polygons by using trigonometry.
- Determine the areas of different triangular regions in the polygons by applying trigonometric knowledge (laws of sines or cosines).
- Solve everyday problems where it is necessary to find the areas of different polygons.

Concepts

- area
- triangular area
- decomposition
- polygons
- triangular regions

Unit 11. Graphs, Polar Equations, and Vectors

On this last unit, we will work with a defined polar coordinate system to find coordinates based on the distance from a fixed point or pole and a rotation or angle relative to a horizontal axis, known as a polar axis. The polar system contains points based on a distance and a rotation angle (r, θ). This angle may be presented in degrees or radians. The lesson demonstrates how to use polar coordinates to work with special equations, where the graphs are curves, as well as with vectors. Vectors represent displacements or movements of objects from one point to another with a certain rotation angle relative to an axis of origin. Polar coordinates may be transformed into Cartesian coordinates and vice versa using the trigonometric functions covered.

At the end of this unit, students will have accomplished the objectives established in the following lessons.

General Objectives

- Define the polar coordinate system and graph points on a polar system.
- Establish the relationship between polar and rectangular coordinates and convert polar coordinates into rectangular coordinates and vice versa.
- Convert an equation given in rectangular coordinates into an equation in polar coordinates and vice versa.
- Define the graph form of a complex number in a coordinate system formed by a real axis and an imaginary axis, and graph complex numbers.
- Define a complex number as $a + bi = z$, where $|z| = \sqrt{a^2 + b^2}$.
- Establish the polar form of a complex number as $z = r(cos\theta + i\ sen\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $tan\ \theta = \frac{b}{a}$, where the value of r is the module of z, and θ is the argument.
- Write complex numbers in their polar form.
- Multiply and divide polar forms of complex numbers.
- Define De Moivre's theorem for powers and roots of complex numbers in their polar form.
- Determine the magnitude of vectors $|v| = \sqrt{a^2 + b^2}$ with their vertical and horizontal displacements: $a = x_2 - x_1\ b = y_2 - y_1$.

Lesson 1. Polar Coordinates and Conversions

Code: C314G0SU11L01

Objectives

- Define the polar coordinates system and find points at a distance and direction from the origin: $\mathrm{P(r, \theta)}$.
- Represent different polar coordinates for a single point.
- Establish the relationship between polar and rectangular coordinates.
- Convert polar coordinates into rectangular coordinates and vice versa.

Concepts

- polar coordinates
- rectangular coordinates
- polar equation
- polar axis
- pole

Lesson 2. Polar Equations and Conversions

Code: C314G0SU11L02

Objectives

- Convert an equation given in rectangular coordinates into an equation in polar coordinates and vice versa.
- Apply trigonometric identities to simplify polar equations.

Concepts

- polar coordinates
- rectangular coordinates
- polar equation
- polar axis
- pole

Lesson 3. Polar Equation Graphs

Code: C314G0SU11L03

Objectives

- Graph a polar equation.

Concepts

- polar coordinate
- rectangular coordinate
- polar equation
- radius

Lesson 4. Trigonometric Functions of Complex Numbers, Vectors, and Applications

Code: C314G0SU11L04

Objectives

- Define the graph form of a complex number in a coordinate system formed by a real axis and an imaginary axis, and graph complex numbers.
- Define a complex number as: $a + bi = z$, where $|z| = \sqrt{a^2 + b^2}$.

- Establish the polar form of a complex number as: $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$. The value of r is the module of z, and θ is the argument.
- Write complex numbers in their polar form.
- Multiply and divide polar forms of complex numbers.
- Define De Moivre's theorem for powers and roots of complex numbers in their polar form.
- Determine the roots of complex numbers.
- Define vector as a line segment or ray with origin and endpoint that represents a quantity with magnitude and direction.
- Define the vector w by its displacement from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$, as $w = \langle x_2 - x_{1,} y_2 - y_1 \rangle$.
- Identify the horizontal displacement: $a = x_2 - x_1$; and the vertical displacement as: $b = y_2 - y_1$.
- Determine the magnitude of vectors with the formula: $|v| = \sqrt{a^2 + b^2}$.
- Define the addition of vectors for $v = \langle a_1, b_1 \rangle$ and $u = \langle a_2, b_2 \rangle$, as: $v + u = \langle a_1 + a_2, b_1 + b_2 \rangle$.
- Define the subtraction of vectors for $v = \langle a_1, b_1 \rangle$ and $u = \langle a_2, b_2 \rangle$, as: $v - u = \langle a_1 - a_2, b_1 - b_1 \rangle$.
- Define the product of a vector using a given scalar $v = \langle a_1, b_1 \rangle$, $cv = c\langle a_1, b_1 \rangle = \langle ca_1, cb_1 \rangle$.
- Define the reference angle to plot vectors on a plane and determine their direction.
- Perform vector operations.

Concepts

- reference angle
- components
- displacement
- direction
- imaginary axis
- real axis
- scalar
- polar form of a complex number
- magnitude
- complex number
- roots of complex numbers
- De Moivre's theorem